유니티(Unity)를 활용한

그래픽스 프로그래밍

03 Geometric Objects-Spaces and Matrix(1)

Spaces

- Vector space
  - The vector space has scalars and vectors.
  - Scalars : $\alpha,\ \beta,\ \delta$□
  - Vectors : u, v, w
- Affine space
  - The affine space has point in addition to the vector space.
  - Points : P, Q, R
- Euclidean space
  - In Euclidean space, the concept of distance is added.

Spaces

Scalars, Points, Vectors

- 3 basic types needed to describe the geometric objects and their relations
  - Scalars: $\alpha, \ \beta, \ \delta$ □ □
  - Points: P, Q, R
  - Vectors: u, v, w
- Vector space
  - scalars & vectors
- Affine space
  - Extension of the vector space that includes a point

Scalars, Points, Vectors

Scalars

- Commutative (교환), associative (결합), and distribution (분배) laws are established for addition and multiplication
  - $\alpha + \beta = \beta + \alpha$
  - $\alpha \cdot \beta = \beta \cdot \alpha$

    commutative (교환)

  - $\alpha + (\beta + \gamma) = (\alpha + \beta) + \gamma$
  - $\alpha \cdot (\beta \cdot \gamma) = (\alpha \cdot \beta) \cdot \gamma$

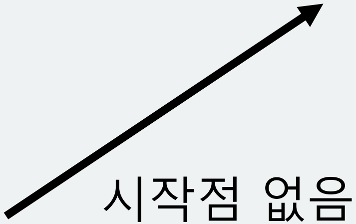
    associative (결합)

  - $\alpha \cdot (\beta + \gamma) = (\alpha \cdot \beta) + (\alpha \cdot \gamma)$ — distribution (분배)
- Addition identity is 0 and multiplication identity is 1.
  - $\alpha + 0 = 0 + \alpha = \alpha$
  - $\alpha \cdot 1 = 1 \cdot \alpha = \alpha$
- Inverse of addition and inverse of multiplication
  - $\alpha + (-\alpha) = 0$
  - $\alpha \cdot \alpha^{-1} = 1$

Scalars

Vectors

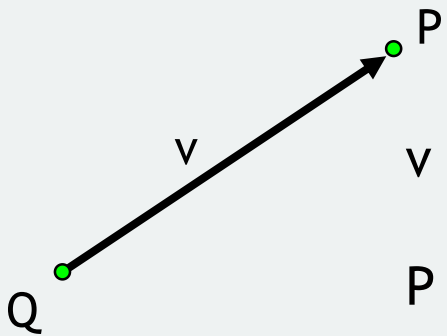
- Vectors have magnitude (or length_크기) and direction (방향).
- Physical quantities, such as velocity or force, are vectors.
- Directed line segments used in computer graphics are vectors.
- Vectors do not have a fixed position in space.



Vectors

Points

- Points have a position in space.
- Operations with points and vectors :
  - Point-point subtraction creates a vector.
  - Point-vector addition creates points.



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$$P = Q + v$$

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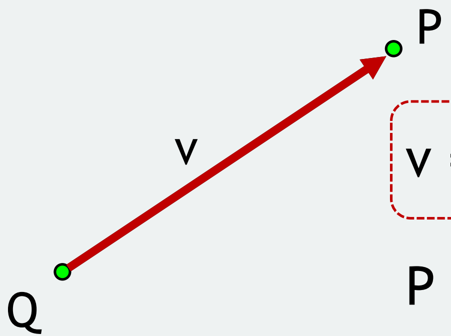

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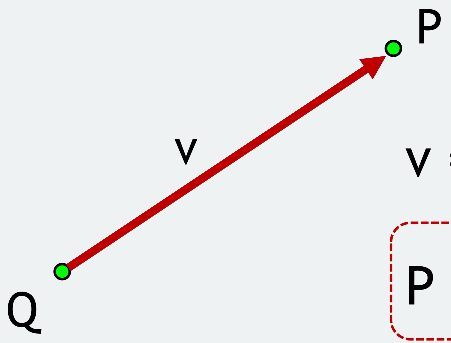

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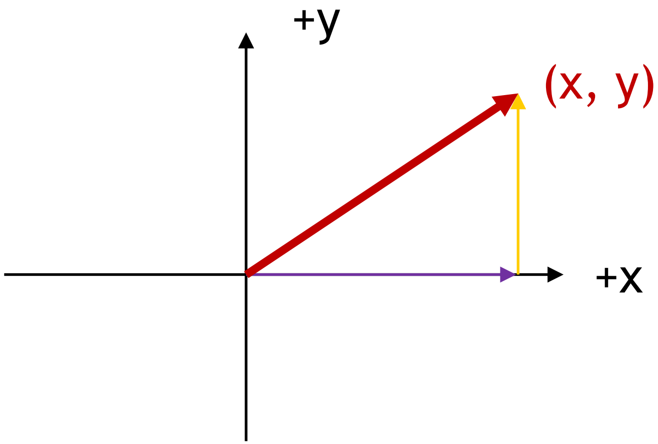
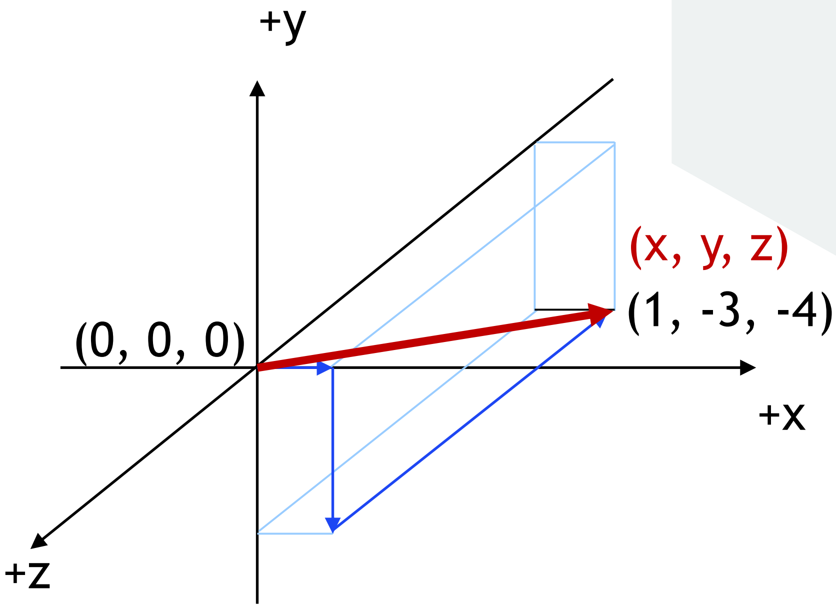
Specifying Vectors

- 2D Vector : (x, y)
- 3D Vector : (x, y, z)



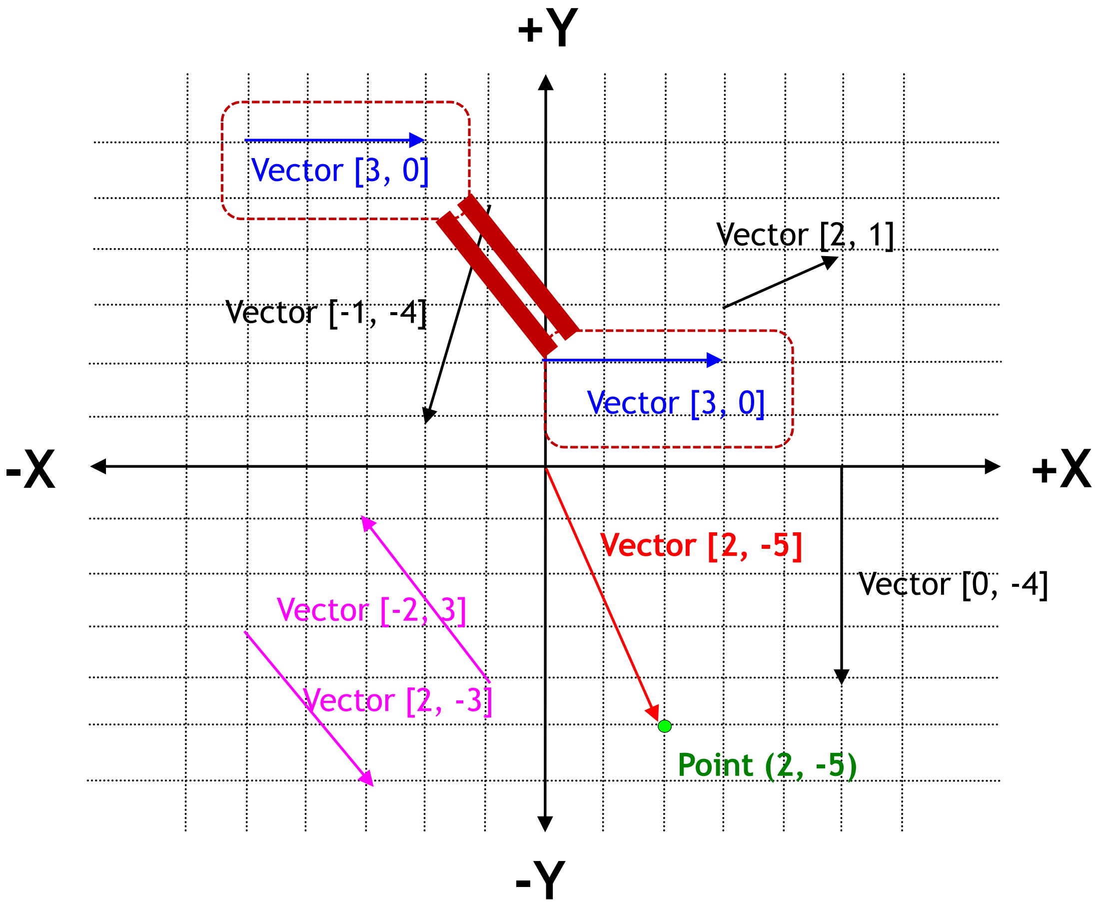
2D Vector



3D Vector

Vector from the origin O(0, 0, 0)
to the point P(1, -3, -4)

Examples of 2D vectors



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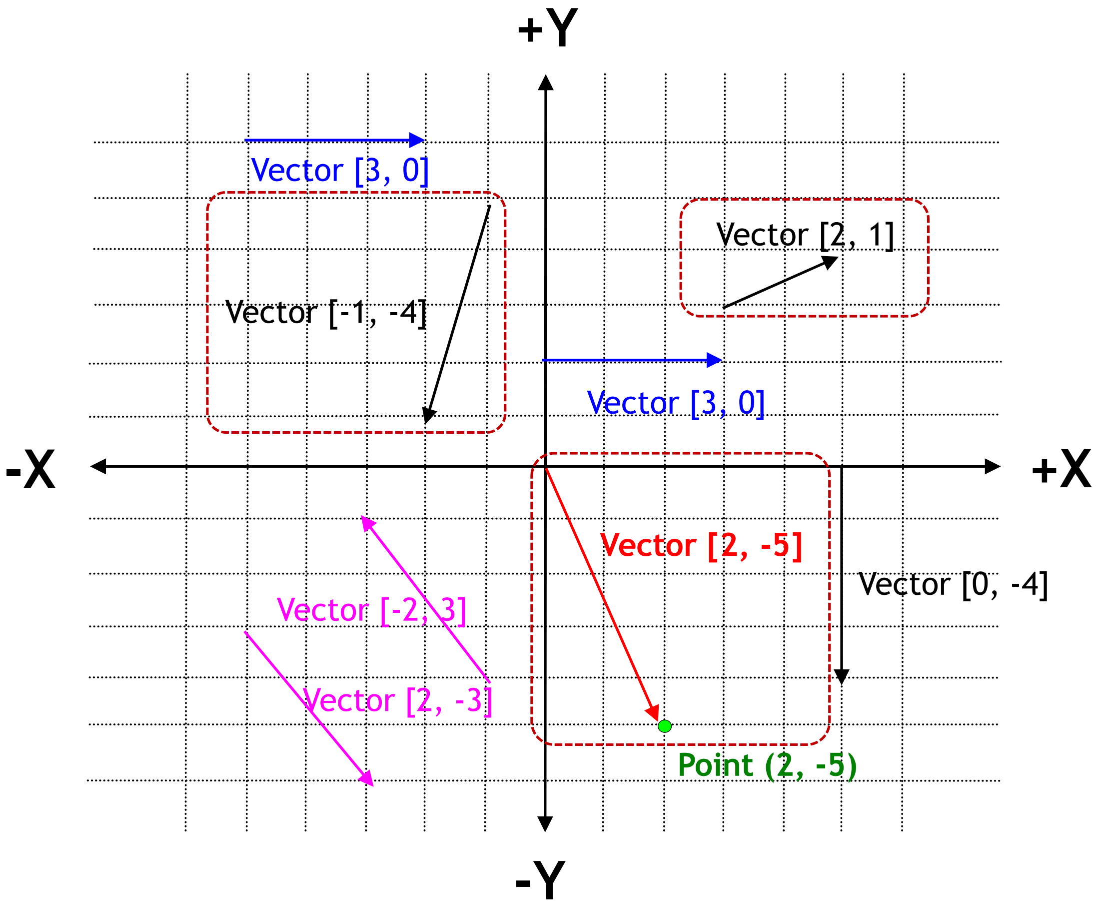

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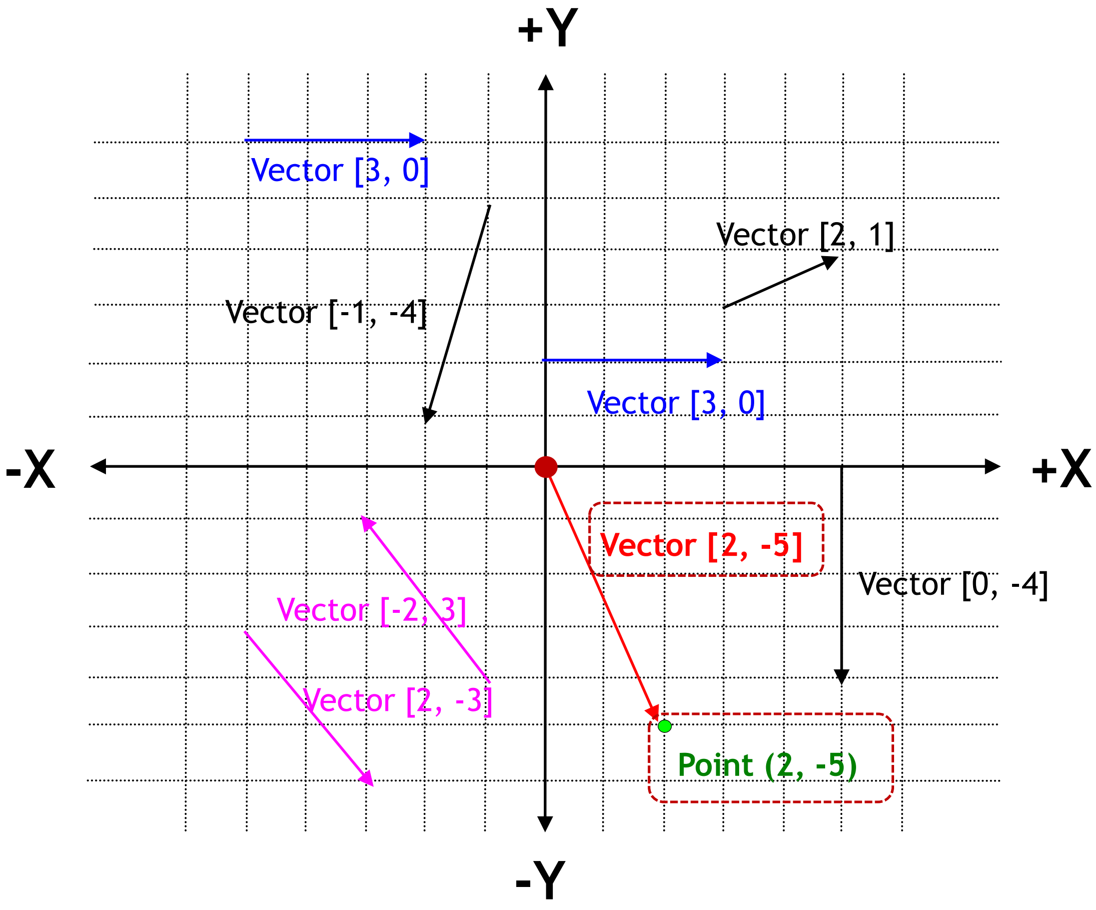

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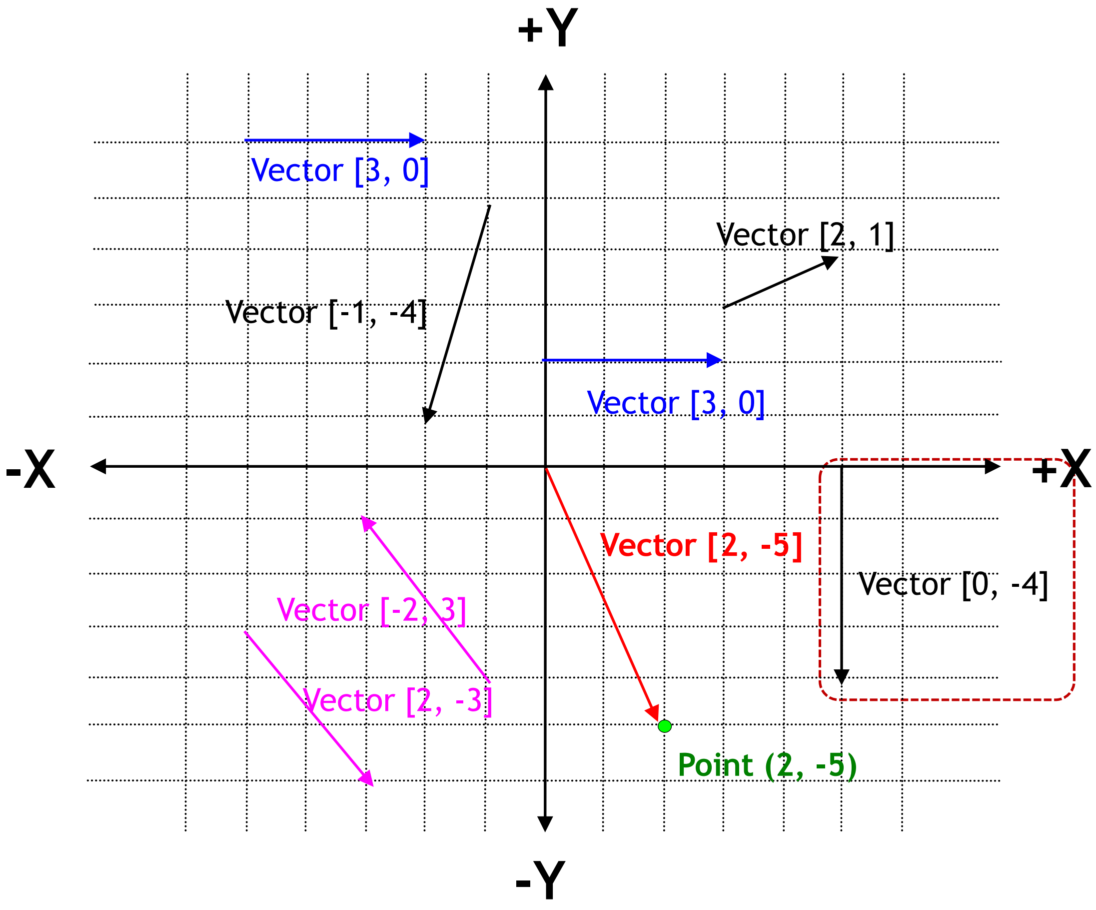

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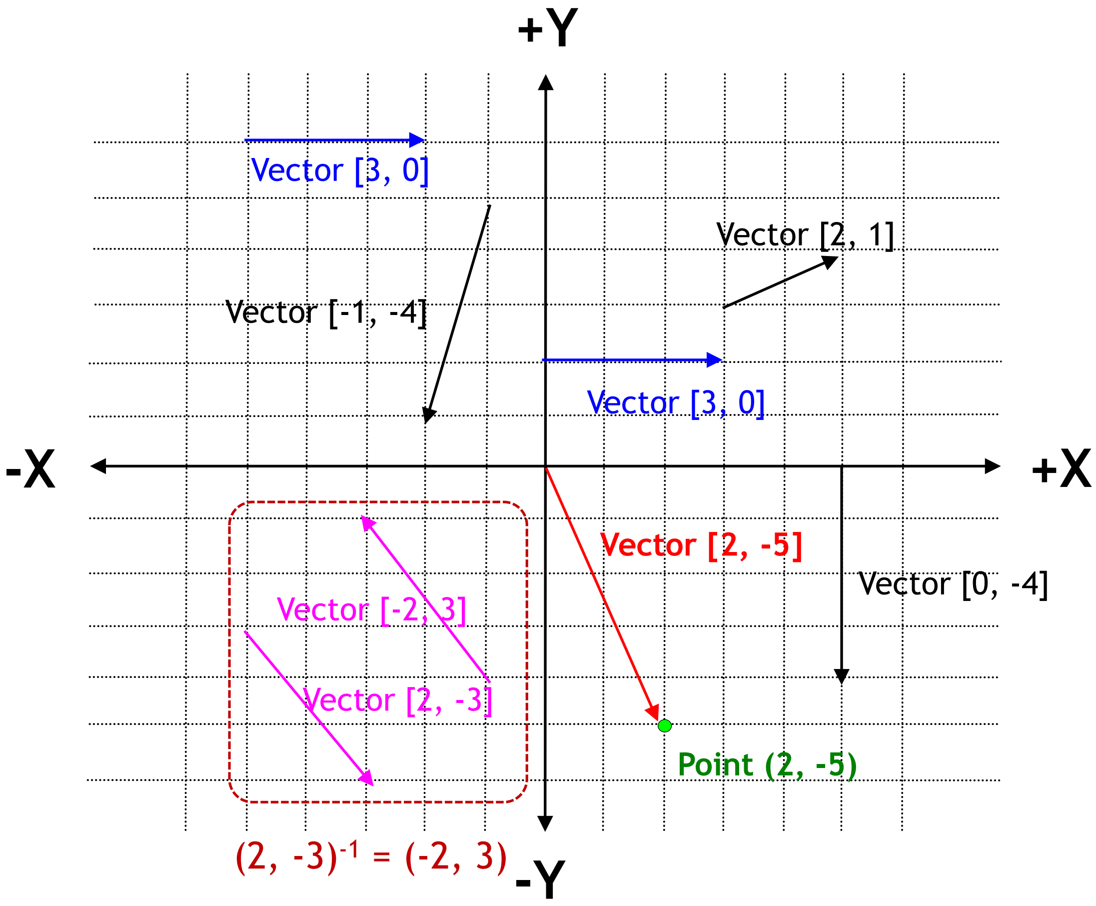


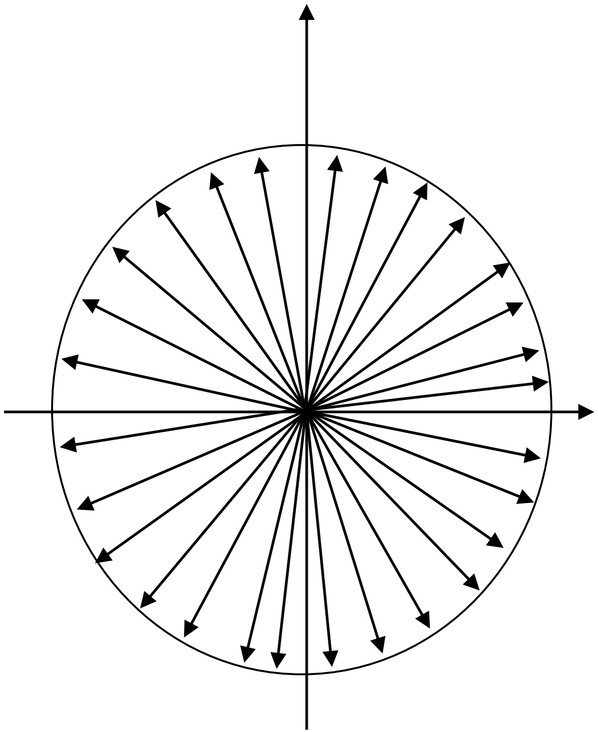
Vector Operations

- zero vector
- vector negation
- vector/scalar multiply
- add & subtract two vectors
- vector magnitude (length)
- normalized vector (=normalization)
- distance formula
- vector product
    - dot product
    - cross product

Vector Operations

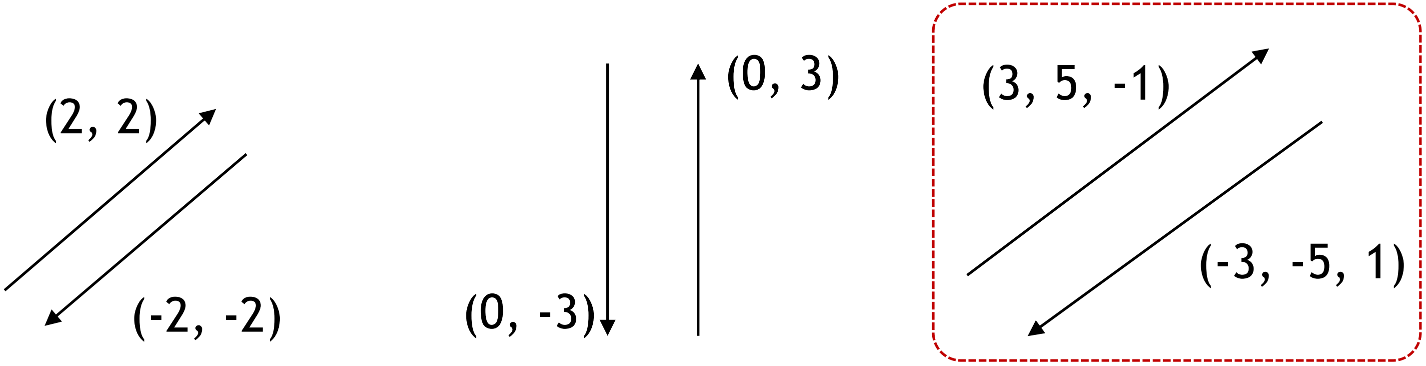
The Zero Vector

- The three-dimensional zero vector is (0, 0, 0).
- The zero vector has zero magnitude.
- The zero vector has no direction.



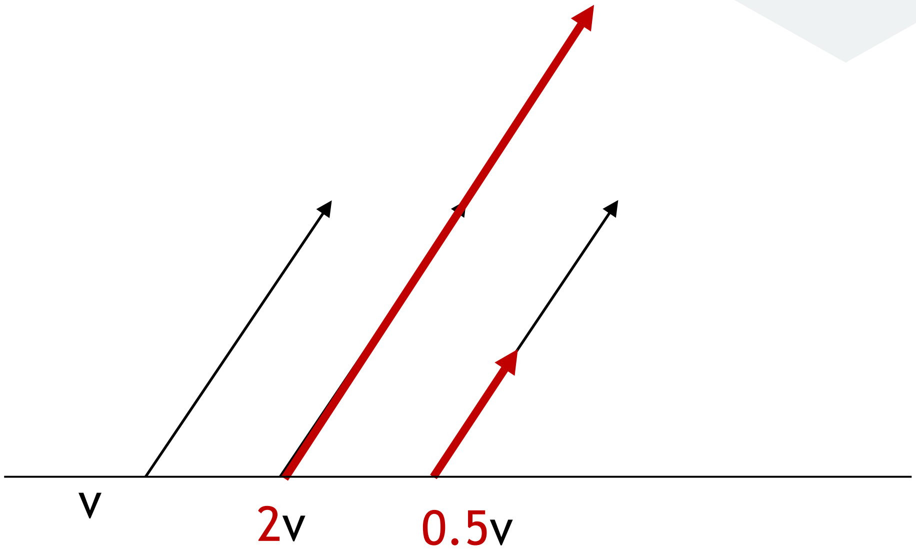
Negating a Vector

- Every vector **v** has a negative vector **–v** : $v + (-v) = 0$
- Negative vector
  - $-(a_1, a_2, a_3, \dots, a_n) = (-a_1, -a_2, -a_3, \dots, -a_n)$
- 2D, 3D, 4D vector negation
  - $-(x, y) = (-x, -y)$
  - $-(x, y, z) = (-x, -y, -z)$
  - $-(x, y, z, w) = (-x, -y, -z, -w)$



Vector-Scalar Multiplication

- Vector scalar multiplication
  - $\alpha * (x, y, z) = (\alpha x, \alpha y, \alpha z)$
- Vector scale division
  - $1/\alpha * (x, y, z) = (x/\alpha, y/\alpha, z/\alpha)$
- Example
  - 2 * (4, 5, 6) = (8, 10, 12)
  - ½ * (4, 5, 6) = (2, 2.5, 3)
  - -3 * (-5, 0, 0.4) = (15, 0, -1.2)
  - $3\mathbf{u} + \mathbf{v} = (3\mathbf{u}) + \mathbf{v}$



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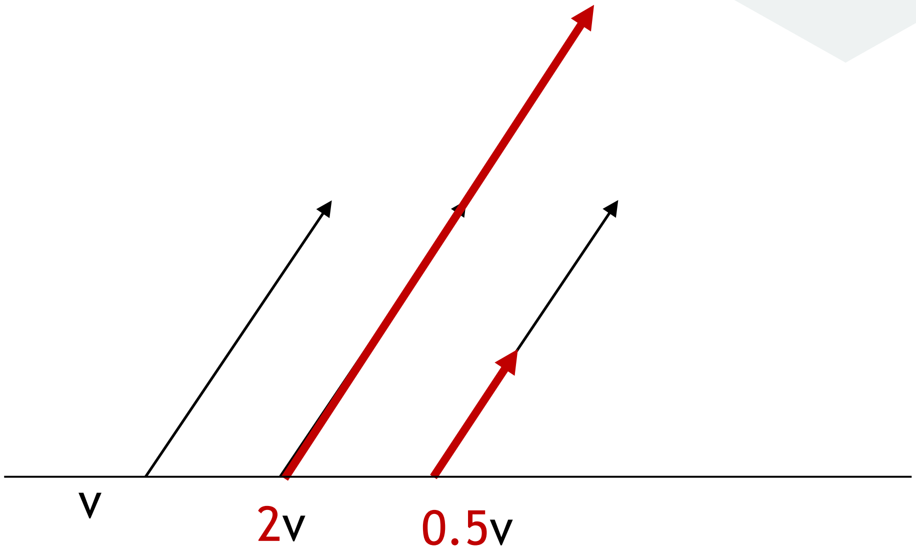

Vector Addition and Subtraction

Vector Addition

- Defined as a head-to-tail axiom

$$(x_1, y_1, z_1) + (x_2, y_2, z_2) = (x_1+x_2, y_1+y_2, z_1+z_2)$$

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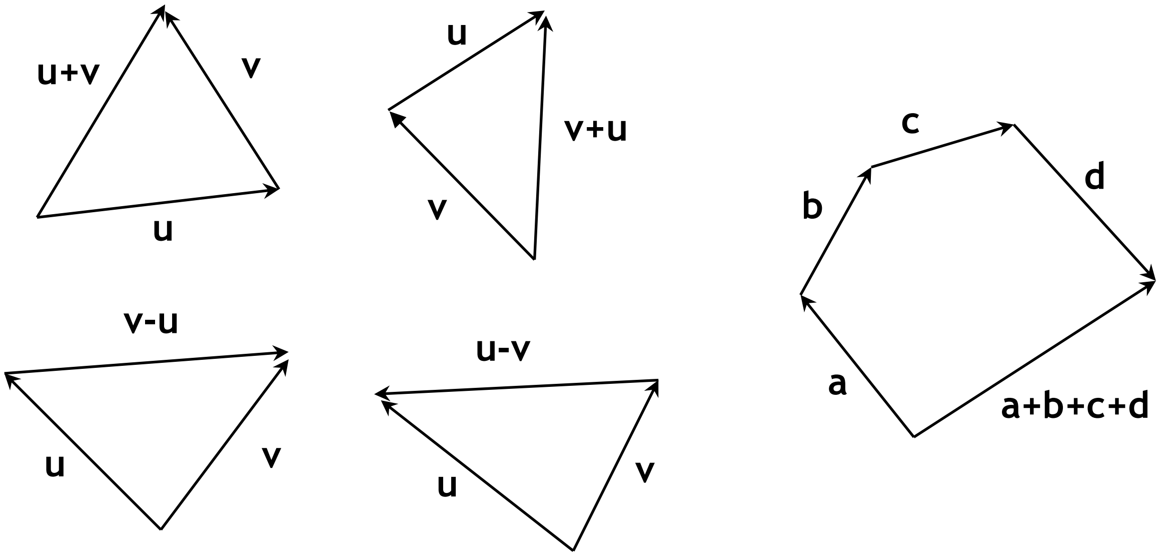

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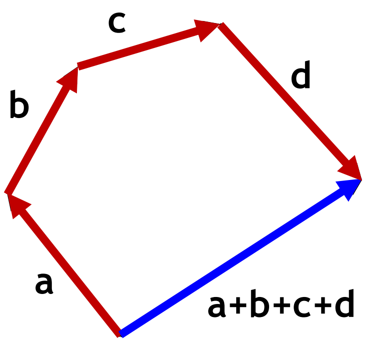
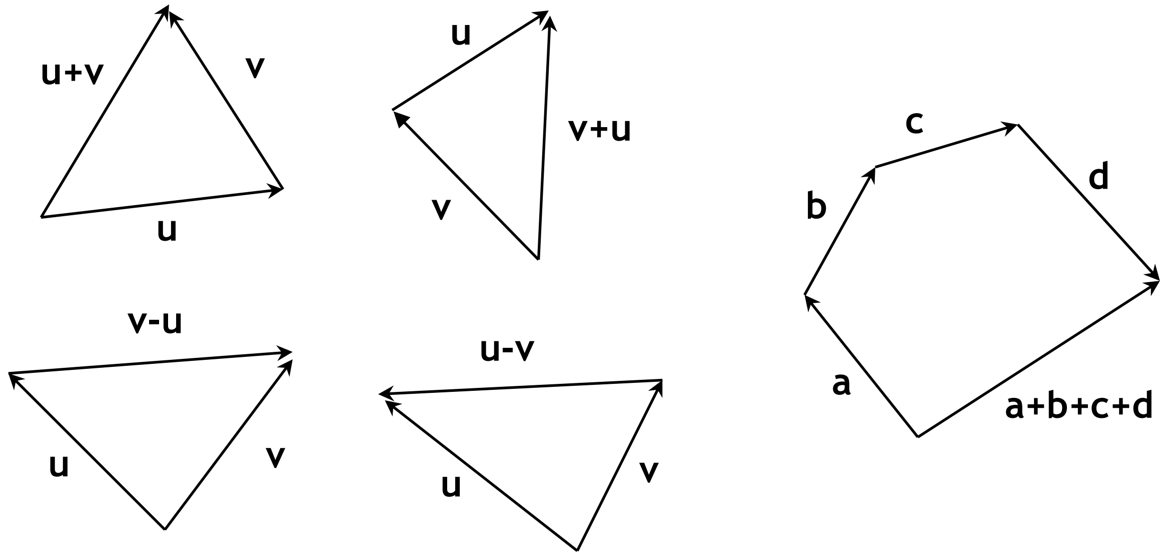


Vector Addition and Subtraction

Vector Subtraction

- $(x_1, y_1, z_1) - (x_2, y_2, z_2) = (x_1-x_2, y_1-y_2, z_1-z_2)$

u - v = -(v - u) – 교환법칙의 역이 성립



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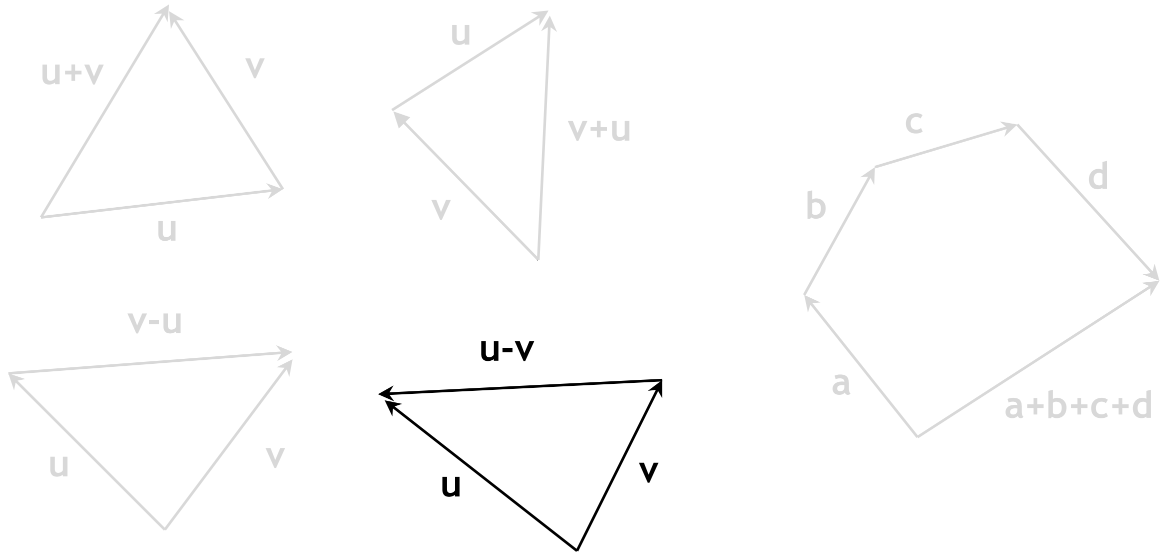

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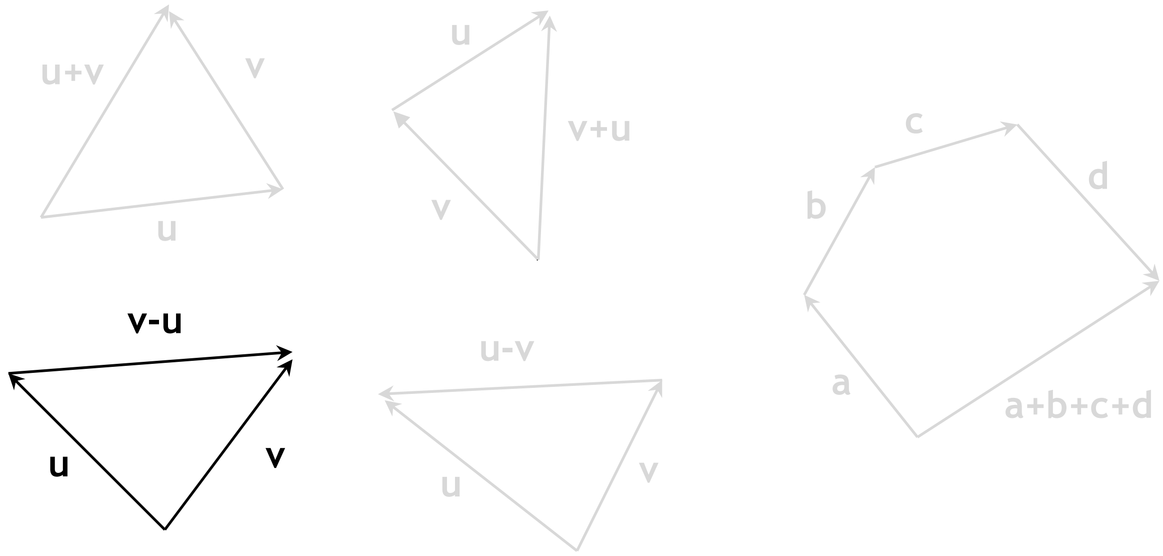

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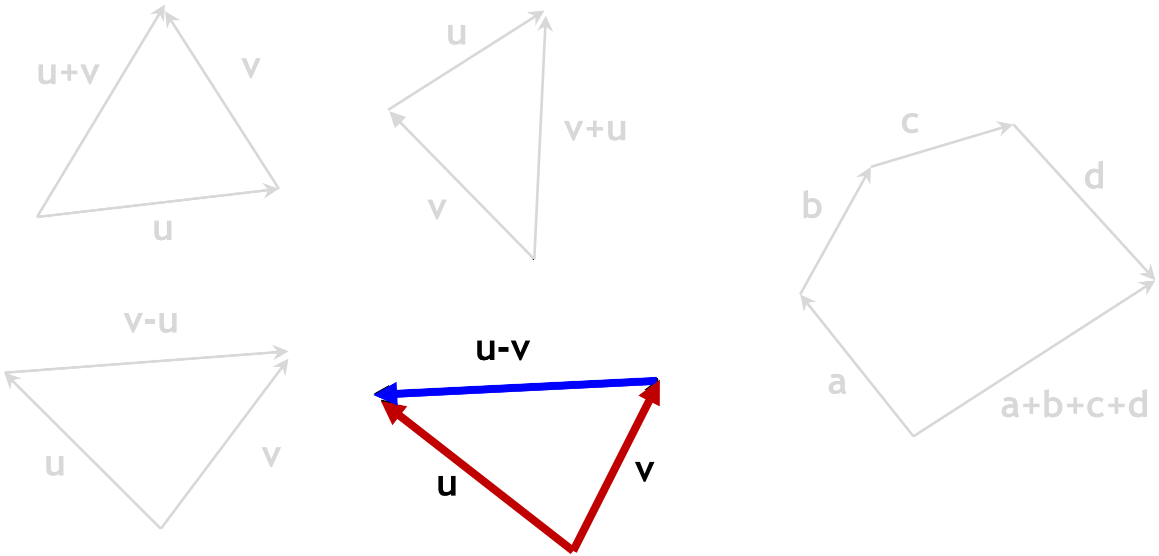

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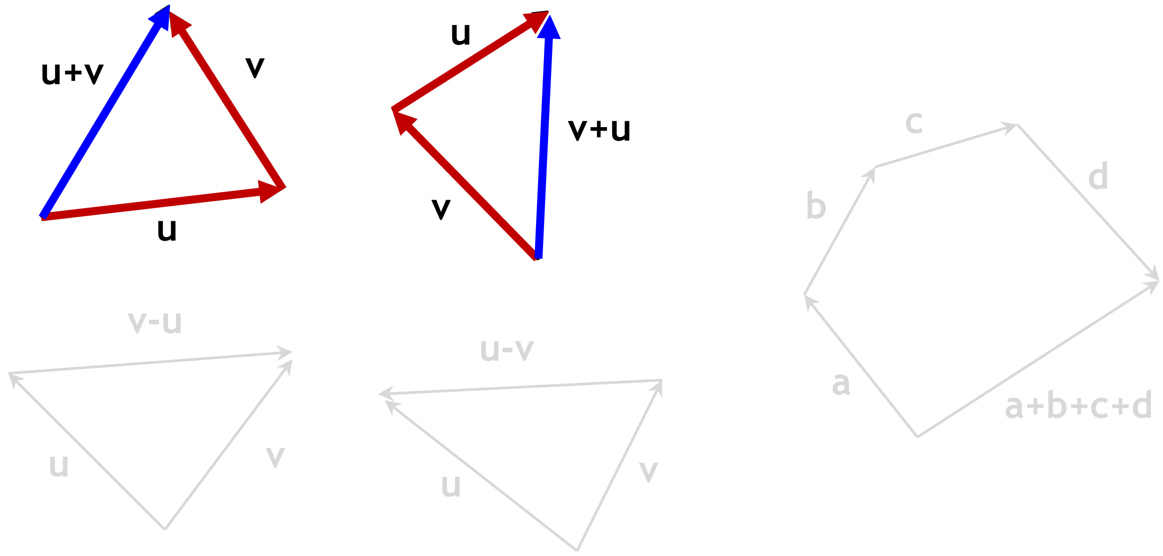
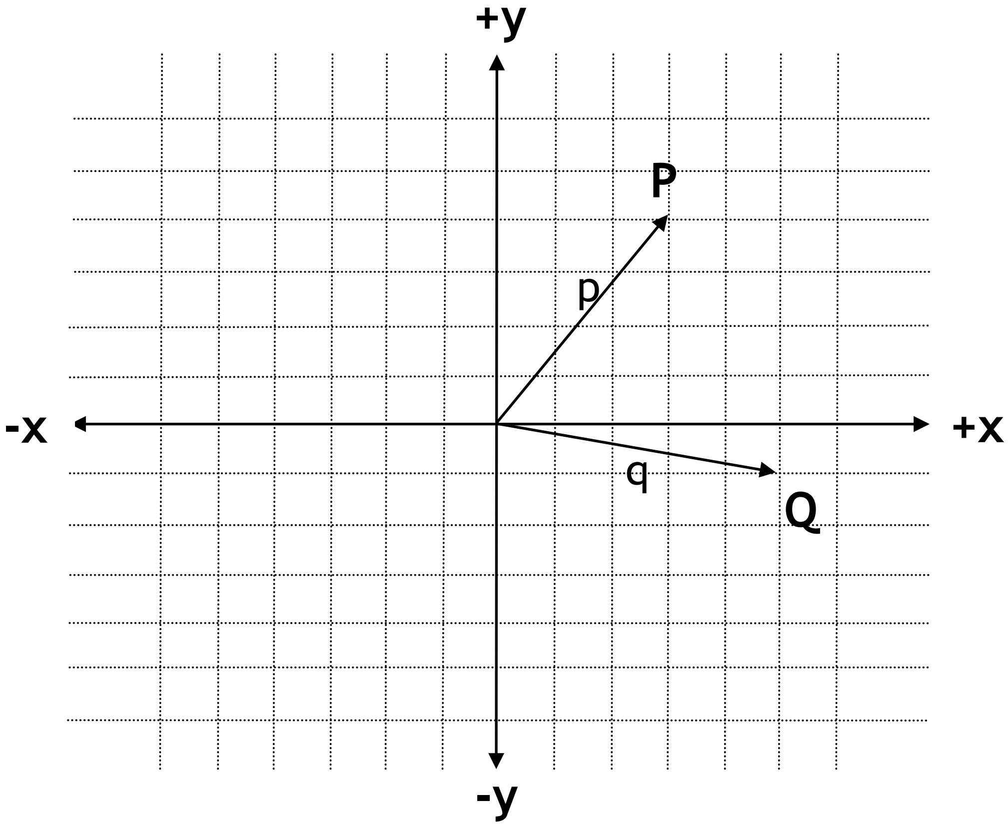


Vector Addition and Subtraction

- The displacement vector from the point P to the point Q is calculated as q – p.



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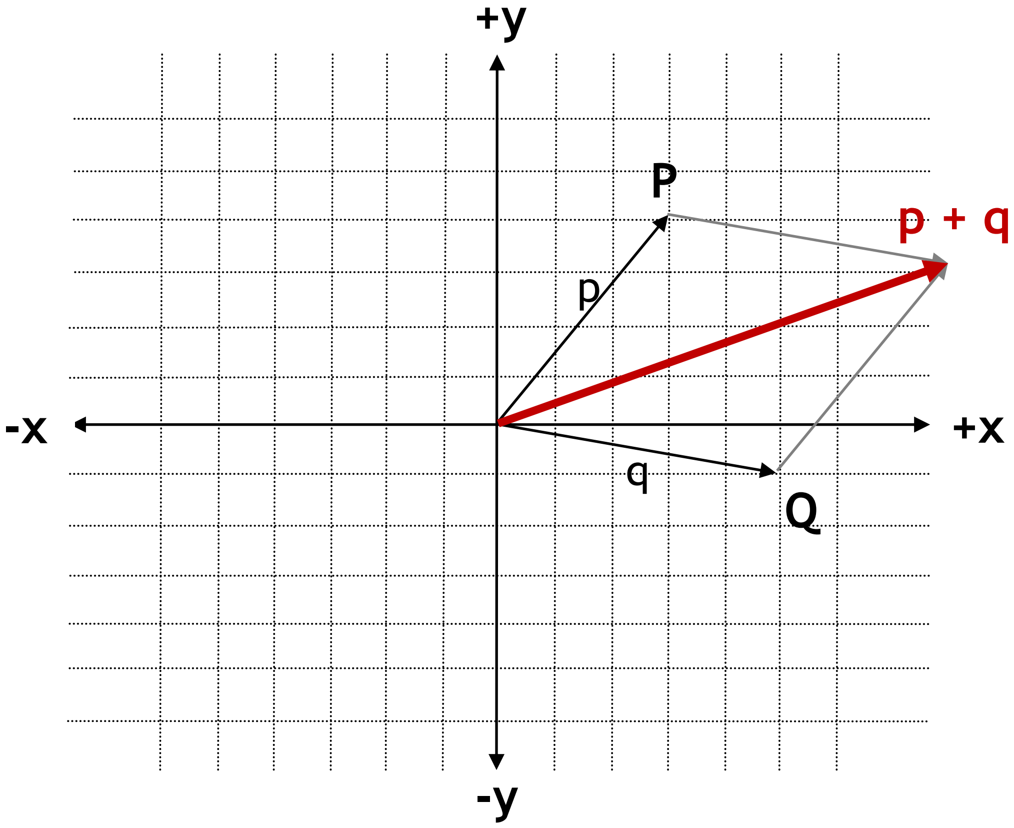

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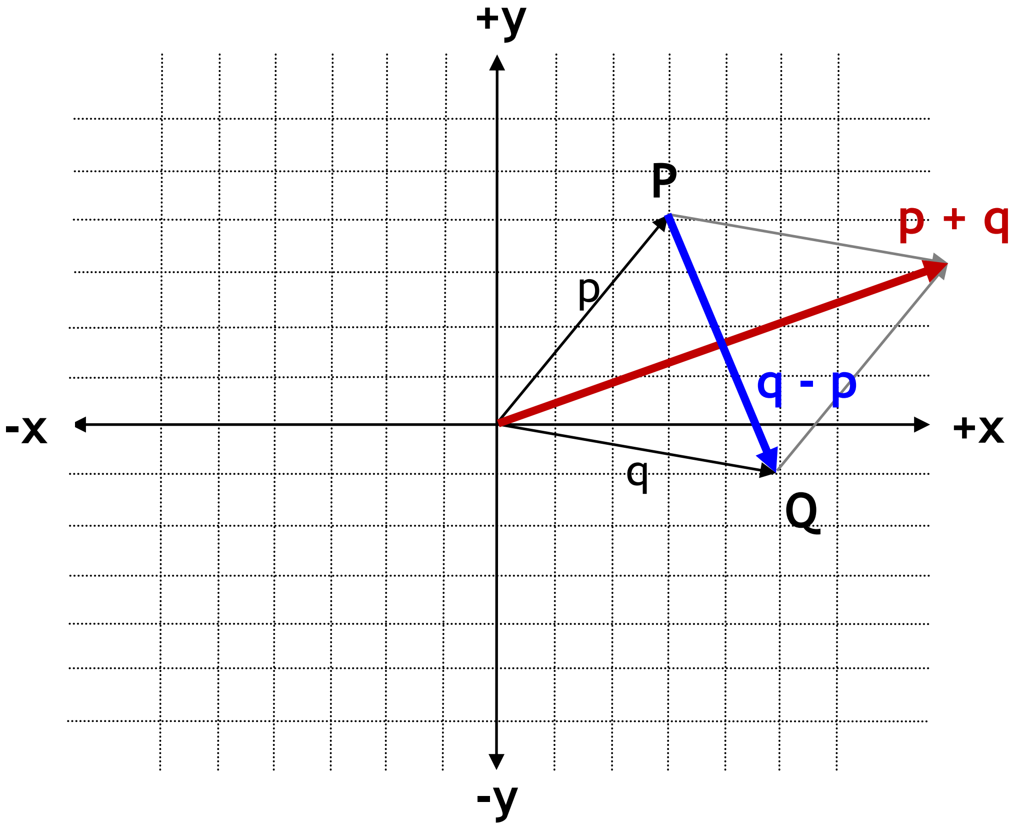
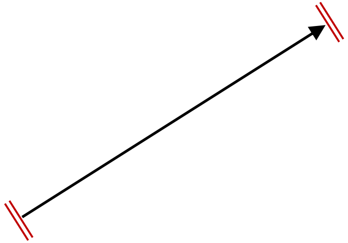


Vector Magnitude(Length)

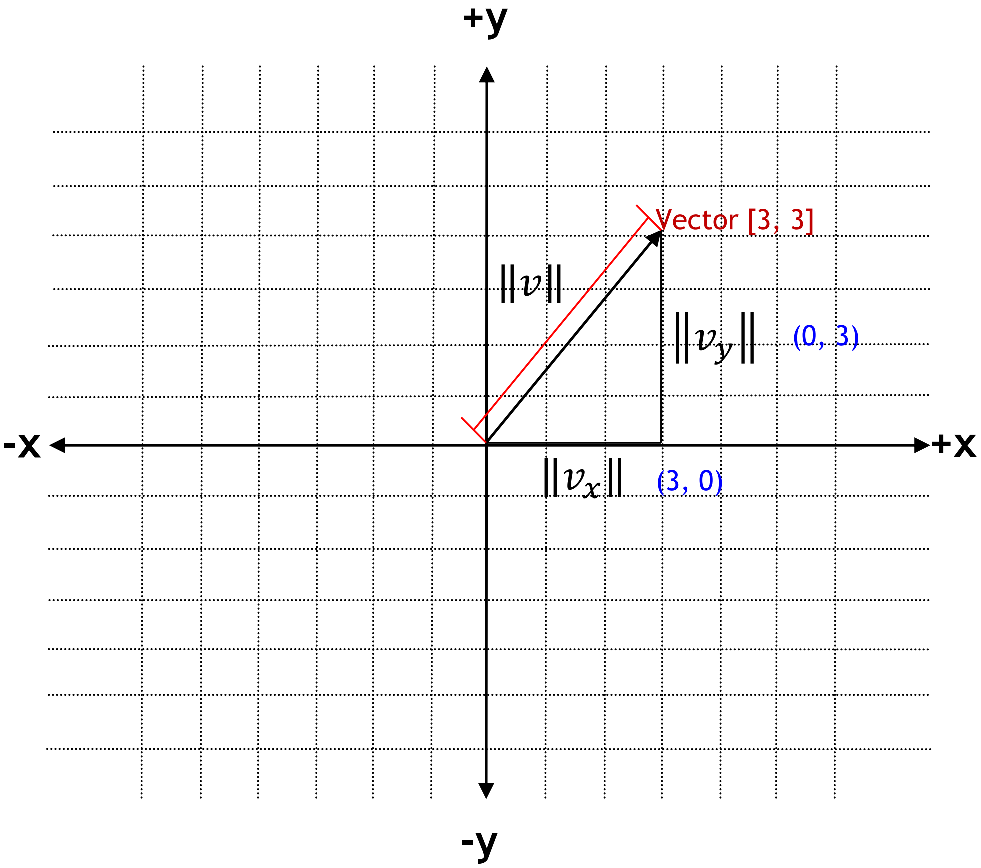
- Vector magnitude(or length)
  - Examples

$$\|v\| = \sqrt{v_1^2 + v_2^2 + \cdots + v_{n-1}^2 + v_{n-2}^2}$$

$$\begin{aligned}\|(5,-4,7)\| &= \sqrt{5^2 + (-4)^2 + 7^2} \\ &= \sqrt{25 + 16 + 49} \\ &= \sqrt{90} \\ &= 3\sqrt{10} \\ &\approx 9.4868\end{aligned}$$



Vector Magnitude



$$\|v\|^2 = |v_x|^2 + |v_y|^2$$

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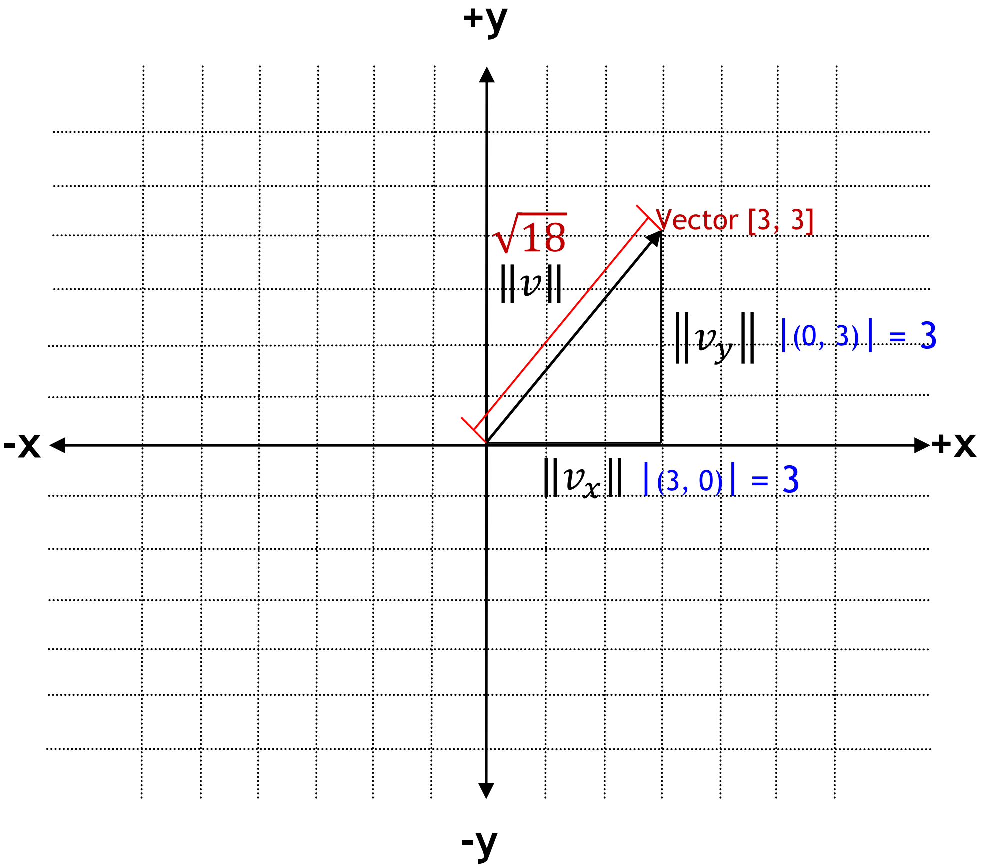
Vector Magnitude



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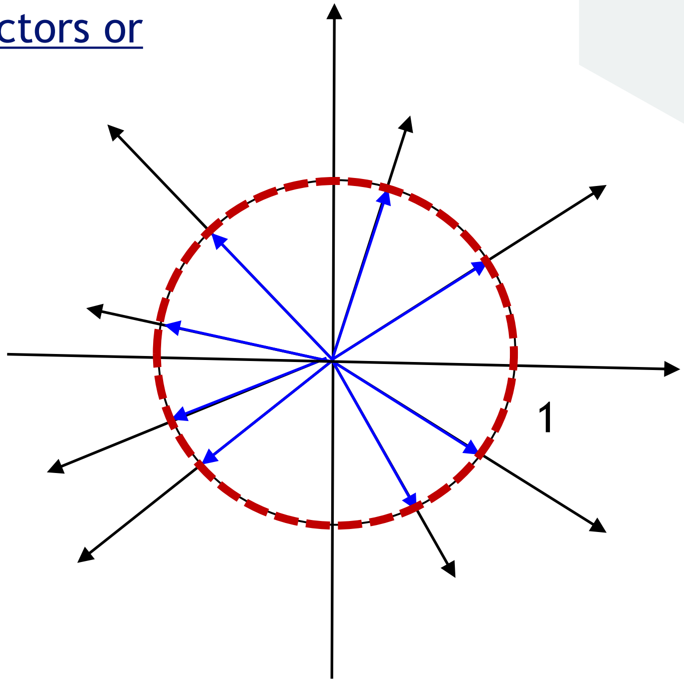
Normalized Vectors

- There is case where you only need the direction of the vector, regardless of the vector length.
- The unit vector has a magnitude of 1.
- The unit vector is also called as normalized vectors or normal.
- “Normalizing” a vector :



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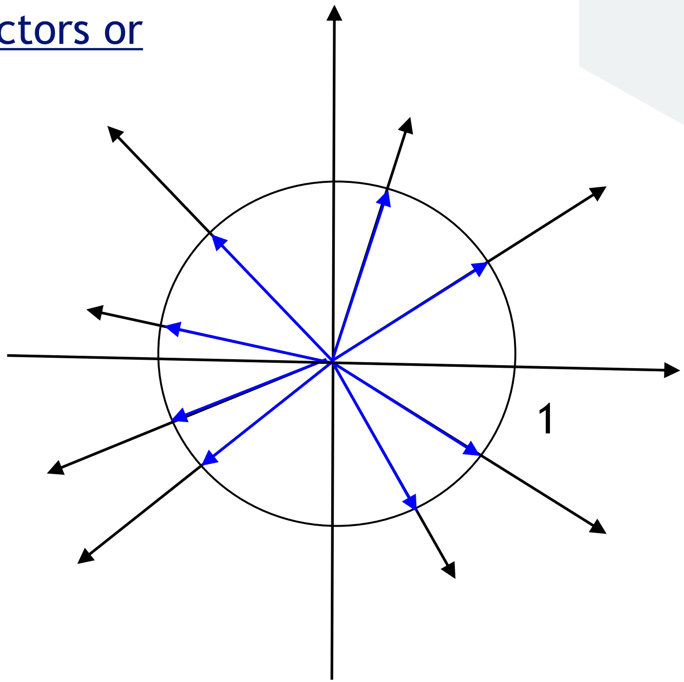
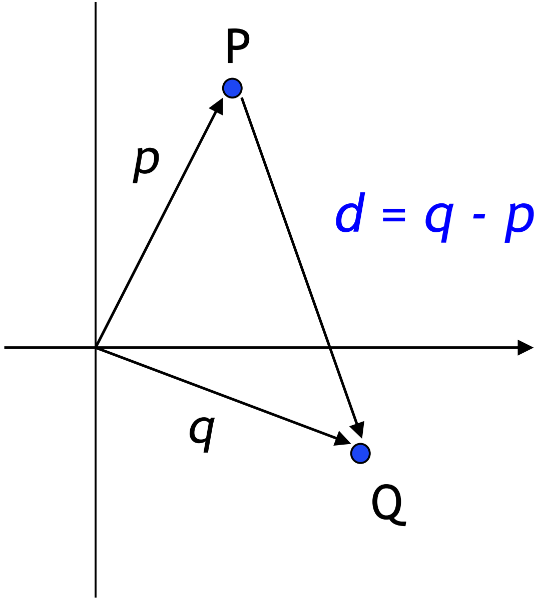
$$v_{norm} = \frac{v}{\|v\|}, \; v \neq 0$$



Distance

- The distance between two points P and Q is calculated as follows.
  - Vector p
  - Vector q
  - Displacement vector d = q - p
  - Find the length of the vector d.
  - distance(P, Q) = ‖ d ‖ = ‖ q - p ‖



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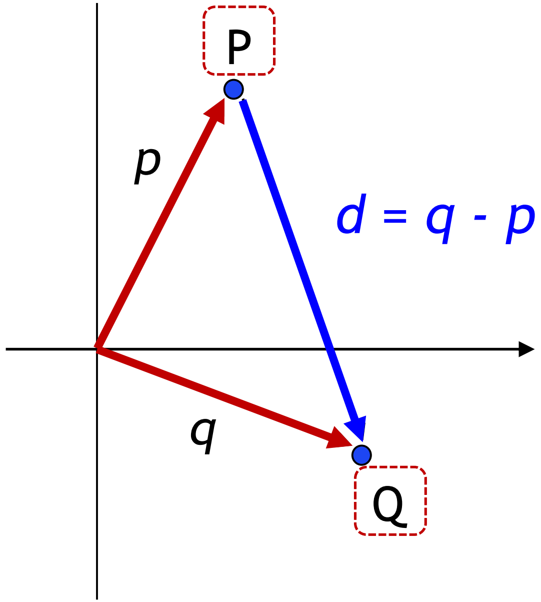


Vector Dot Product

- Dot product between two vectors : u•v = scalar

$$(u_1, u_2, u_3, \dots, un) \cdot (v_1, v_2, v_3, \dots, vn) = u_1v_1 + u_2v_2 + \dots + u_{n-1}v_{n-1} + u_nv_n$$

$$u \bullet v = \sum_{i=1}^{n} u_i v_i$$

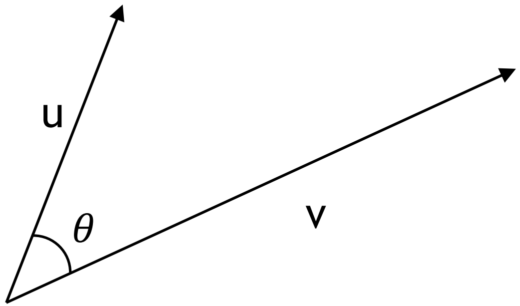
$$u \bullet v = \|u\|^2$$

- Example
  - (4, 6)•(-3, 7) = 4 * (-3) + 6 * 7 = 30
  - (3, -2, 7)•(0, 4, -1) = 3 * 0 + (-2) * 4 + 7 * (-1) = -15

Vector Dot Product

- The dot product of the two vectors is the cosine of the angle between two vectors (assuming they are normalized).

$$u \bullet v = \|u\|\|v\|cos\theta$$



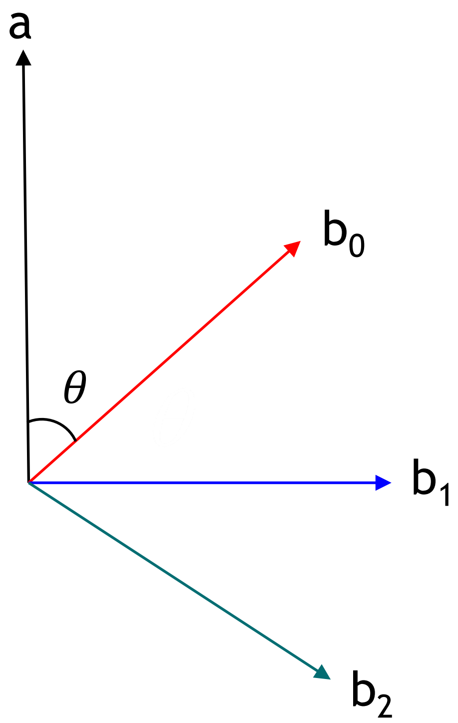
$$\theta = \mathrm{acos}(\frac{u \bullet v}{\|u\|\|v\|})$$

$$\theta = \mathrm{acos}(u \bullet v)$$, where u, v are unit vectors

length= 1

Dot Product as Measurement of Angle

- The following is the characteristics of the dot product.



$a \bullet b_0 > 0 \; when \; 0° \le \theta < 90°$ (양수일 때, 예각)

$a \bullet b_1 = 0 \; when \; 0° = \theta = 90°$ (90° 일 때, 직교)

$a \bullet b_2 < 0 \; when \; 90° < \theta \le 90°$ (음수일 때, 둔각)

Projecting One Vector onto Another

- Given two vectors, w and v, one vector w can be divided into parallel and orthogonal to the other vector v.

$$w = w_{par} + w_{per}$$

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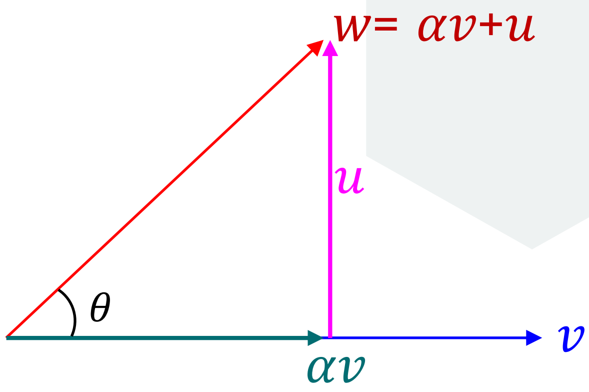

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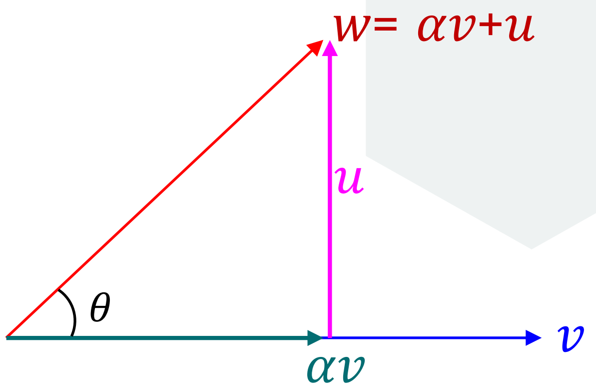

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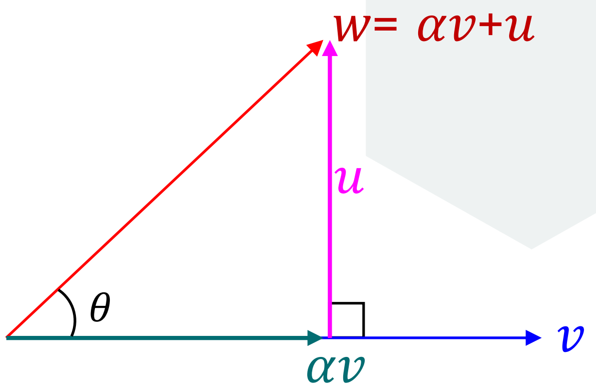

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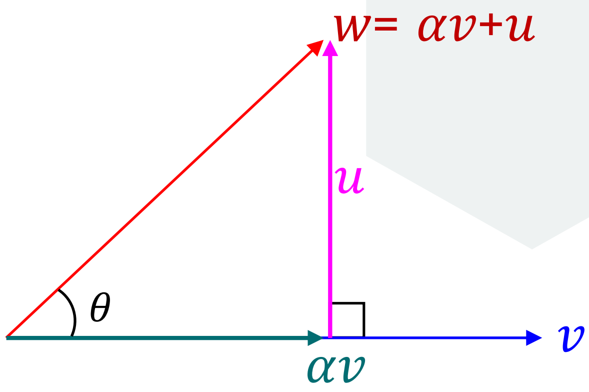

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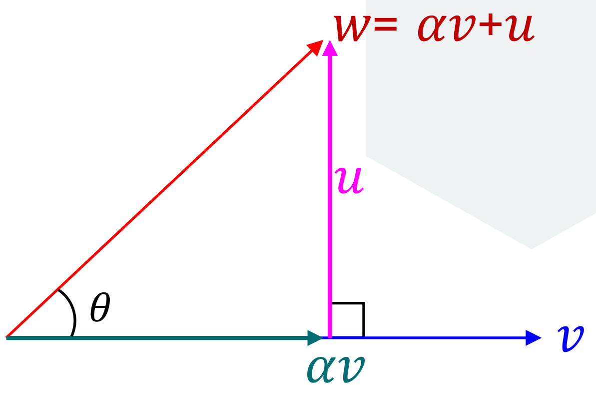
$$\alpha = \frac{w \cdot v}{v \cdot v}$$



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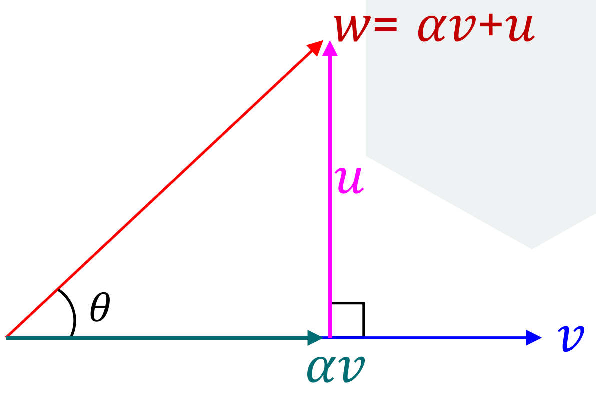
$$\alpha = \frac{w \cdot v}{v \cdot v}$$

$$u = w - \alpha v = w - \frac{w \cdot v}{v \cdot v} v = w - \frac{w \cdot v}{||v^2||} v$$



Projecting One Vector onto Another

- Given two vectors, w and v, one vector w can be divided into parallel and orthogonal to the other vector v.

$$w = w_{par} + w_{per}$$

$$w = \alpha v + u$$

$$u \text{ must be orthogonal to } v,\ u \cdot v = 0$$

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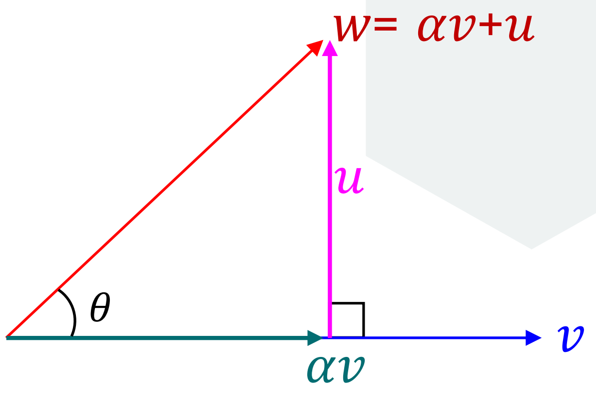


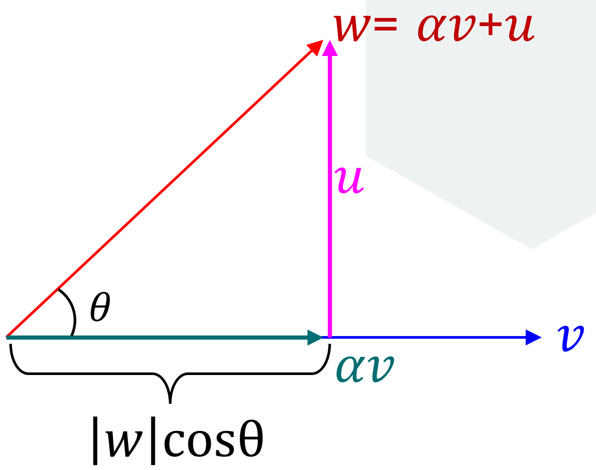
Projecting One Vector onto Another

- Given two vectors, w and v, one vector w can be divided into parallel and orthogonal to the other vector v.

$$If\ v\ is\ a\ unit\ vector, then\ \|v\| = 1$$

$$w_{per} = u = w - (w \cdot v)v$$

= 1



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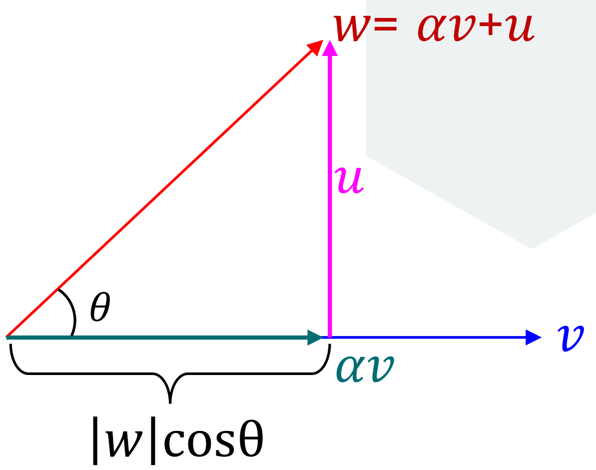

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$$cos\theta = \frac{\|\alpha v\|}{\|w\|} \Rightarrow \|\alpha v\| = \|w\|cos\theta$$

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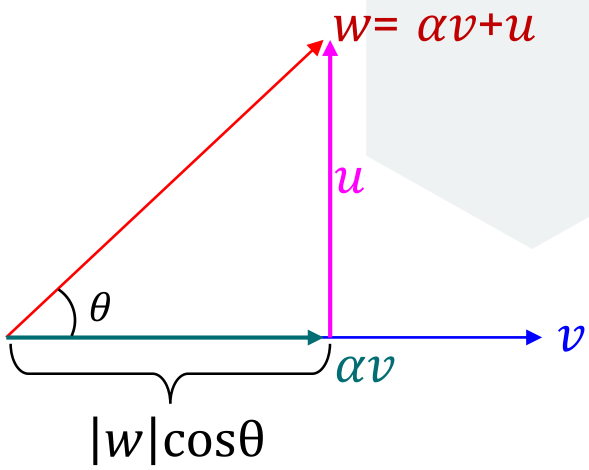
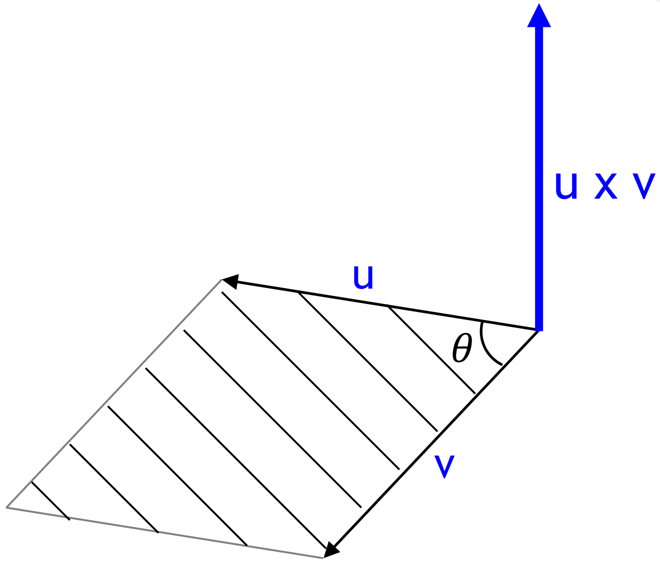


Vector Cross Product

- Cross product (외적) : u x v
  - $(x_1, y_1, z_1) \times (x_2, y_2, z_2) = (y_1 z_2 - z_1 y_2, z_1 x_2 - x_1 z_2, x_1 y_2 - y_1 x_2)$
- Dot Product (내적) : u•v



Vector Cross Product

- Cross product (외적) : u x v = w
  - $(x_1, y_1, z_1) \times (x_2, y_2, z_2) = (y_1z_2 - z_1y_2, z_1x_2 - x_1z_2, x_1y_2 - y_1x_2)$
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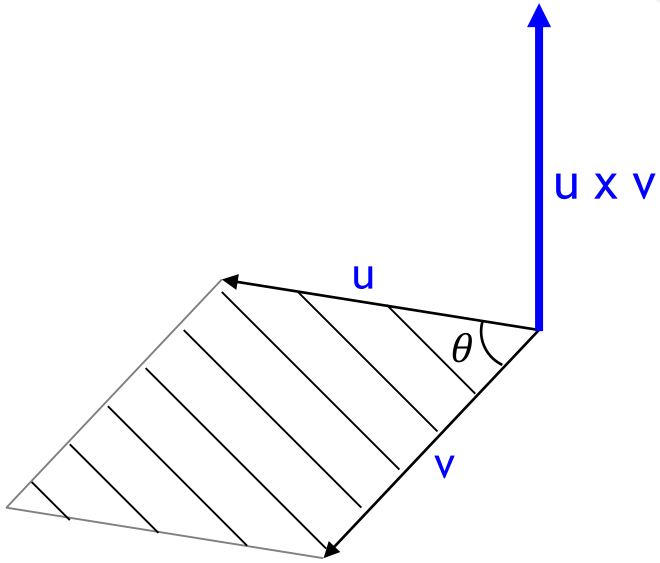

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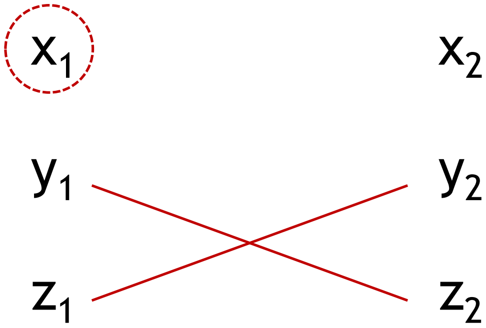

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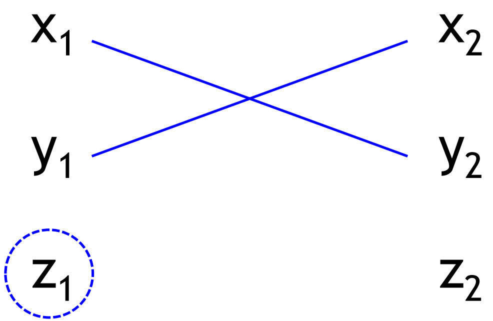

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    $= -x_1z_2 + z_1x_2$
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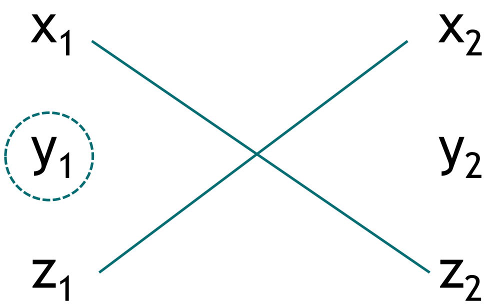

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- Dot Product (내적) : u•v = α
- Example
  - (1, 3, -4) x (2, -5, 8)

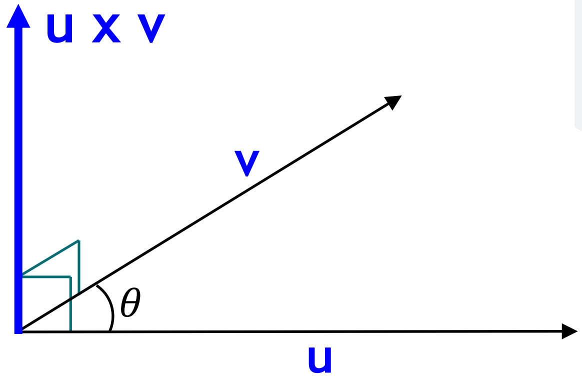
    = (3 * 8 - (-4) * (-5), (-4) * 2 - 1 * 8, 1 * (-5) – 3 * 2)

    = (4, -16, -11)

Vector Cross Product

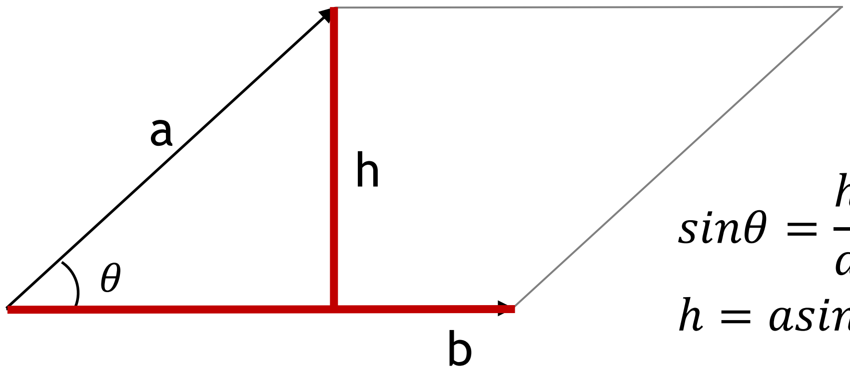
- The magnitude of the cross product between two vectors, |(u x v)|, is the product of the magnitude of each other and the sine of the angle between the two vectors.

$$\|u \times v\| = \|u\|\|v\|sin\theta$$



Vector Cross Product

- The area of the parallogram is calculated as bh.



$$\sin\theta = \frac{h}{a}$$

$$h = a\sin\theta$$

$$A = bh$$

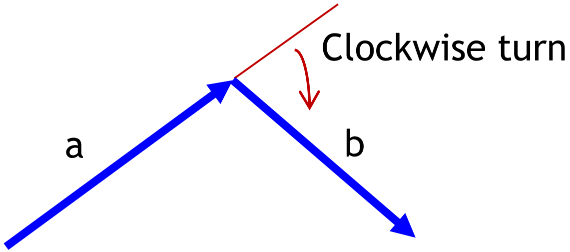
$$= b(a\sin\theta)$$

$$= \|a\|\|\mathrm{b}\|\sin\theta$$

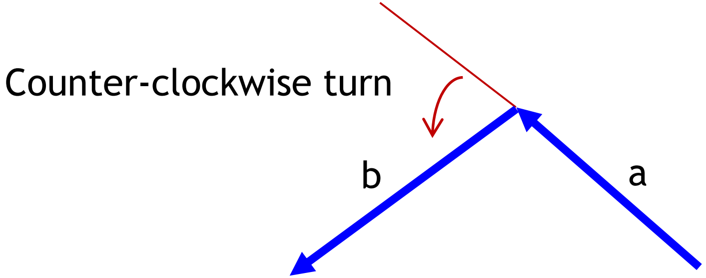
$$= \|a \times \mathrm{b}\|$$

Vector Cross Product

- In the left-handed coordinate system, when the vectors u and v move in a clockwise turn, u x v points in the direction toward us, and when moving in a counter-clockwise turn, u x v points in the direction away from us.
- In the right-handed coordinate system, when the vectors u and v move in a counter-clockwise (반시계) turn, u x v points in the direction toward us, and when moving in a clockwise turn, u x v points in the direction away from us.



Left-handed Coordinates



Right-handed Coordinates

Linear Algebra Identities

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$u + v = v + u$	Vector addition commutative law
$u - v = u + (-v)$	Vector subtraction
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$\alpha(\beta u) = (\alpha\beta)u$	Scalar-Vector multiplication association
$\alpha(u + v) = \alpha u + \alpha v$ $(\alpha + \beta)u = \alpha u + \beta u$	Scalar-Vector distribution law
$\|\alpha v\| = \|\alpha\|\|v\|$	Scalar product
$\|v\| \geq 0$	The magnitude of vector is nonnegative
$\|u\|^2 + \|v\|^2 = \|u + v\|^2$	Pythagorean theorem
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$\alpha(u \cdot v) = (\alpha u) \cdot v = u \cdot (\alpha v)$	Vector dot product and scalar product associative law
$u \cdot (v + w) = u \cdot v + u \cdot w$	Vector addition and dot product distribution law
$u \times u = 0$	Cross product of the vector itself is 0.
$u \times v = -(v \times u)$	Cross product is anti-commutative.
$u \times v = (-u) \times \_(-v)$	Cross product of a vector is equal to the cross product of inverse of each vector.
$\alpha(u \times v) = (\alpha u) \times v = u \times (\alpha v)$	Scalar and cross product multiplication associative law
$u \times (v + w) = (u \times v) + (u \times w)$	Cross product of vector and the addition of two vector establish the distribution law
$u \cdot (u \times v) = 0$ $v \cdot (u \times v) = 0$	Dot product of any vector with cross product of that vector & another vector is 0

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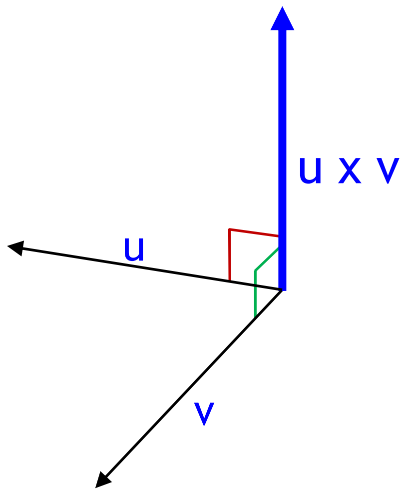

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Geometric Objects

- Line
  - 2points
- Plane
  - 3points
- 3D objects
  - Defined by a set of triangles
  - Simple convex flat polygons (볼록다각형)
  - hollow (비어 있음)

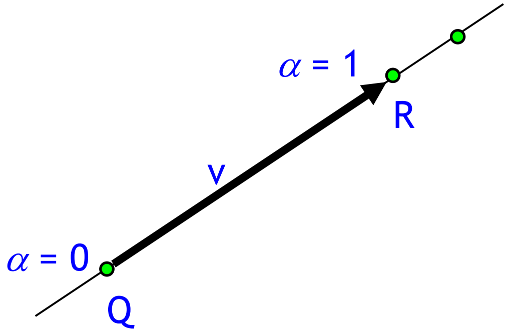
Geometric Objects

Lines

- Line is point-vector addition (or subtraction of two points).
- Line parametric form : $P(\alpha) = P_0 + \alpha v$
  - P_0 is arbitrary point, and v is arbitrary vector
  - Points are created on a straight line by changing the parameter.

$$v = R - Q$$

$$P = Q + \alpha v = Q + \alpha(R - Q) = Q + \alpha R - \alpha Q = \alpha R + (1 - \alpha)Q$$



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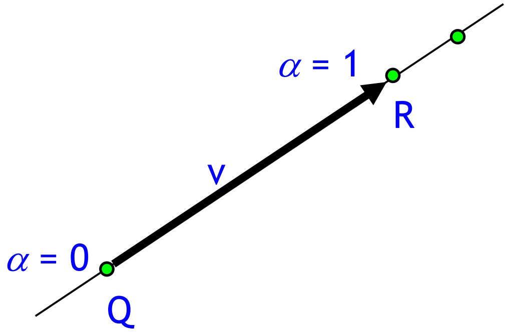

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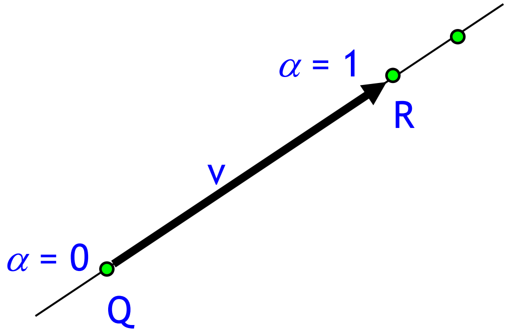
$$P = \alpha_1 R + \alpha_2 Q \;\; where \;\; \alpha_1 + \alpha_2 = 1$$

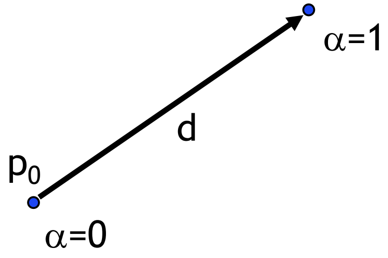
$\alpha_1 = \alpha$, $\alpha_2 = 1 - \alpha$

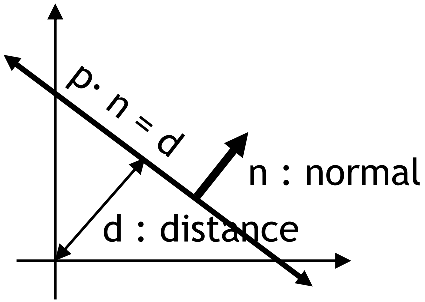


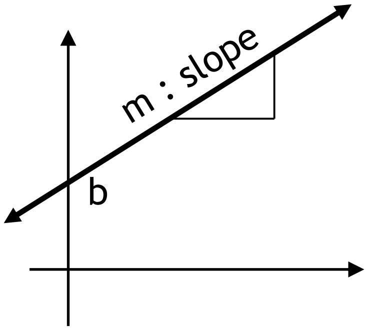
Lines, Rays, Line Segments

- The line is infinitely long in both directions.
- A line segment is a piece of line between two endpoints. 0 <= α <= 1
- A ray has one end point and continues infinitely in one direction. α >= 0
- Line
  - $p(\alpha) = p_0 + \alpha d$ (parametric)
  - $y = mx + b$ (explicit)
  - $ax + by = d$ (implicit)
  - $p \cdot n = d$

    $(x, y) \cdot (a, b) = d$







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