Viewing

Fall 2021
11/02/2021
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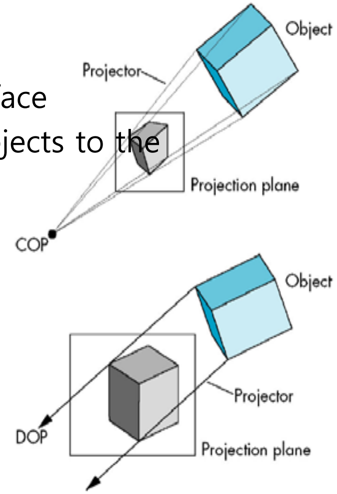
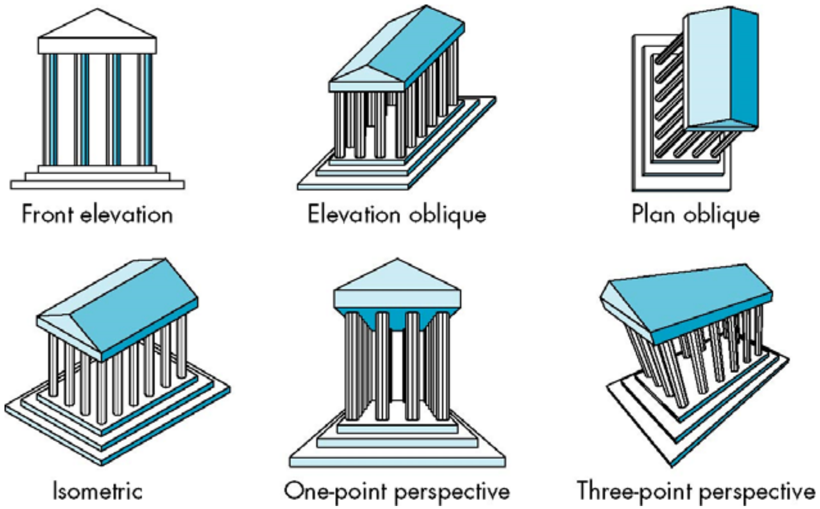
Viewing

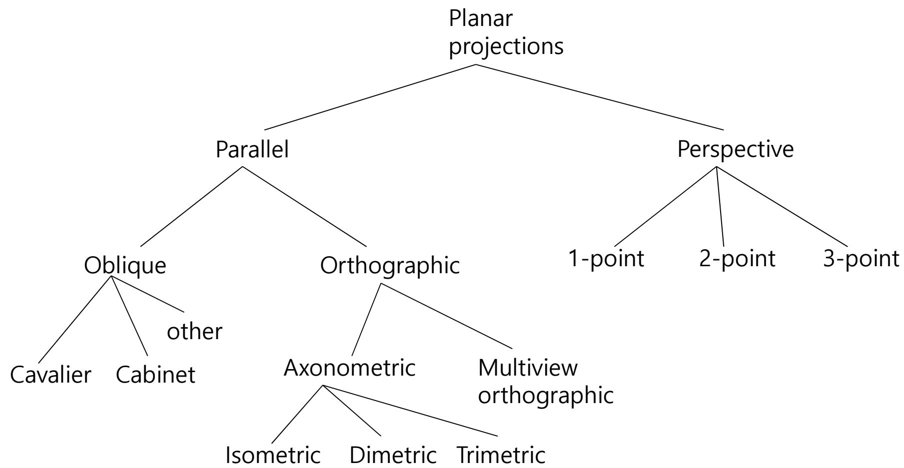
- Viewing requires basic elements
  - One or more objects
  - A viewer with a projection surface
  - Projectors that go from the objects to the projection plane
- COP vs DOP
  - Center Of Projection (COP)
  - – Perspective views
  - Direction Of Projection (DOP)
  - – Parallel views



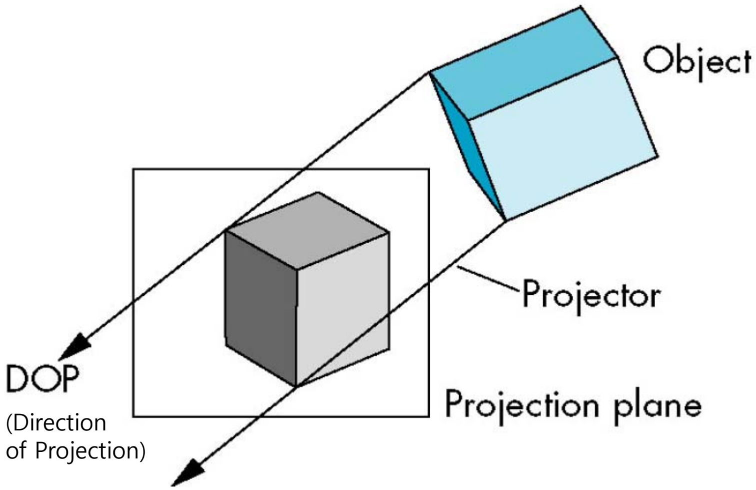
Classical Viewing



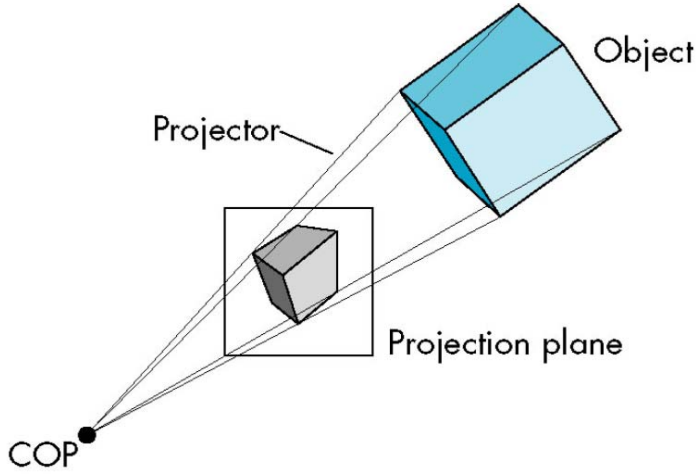
Classical Viewing



Parallel Viewing

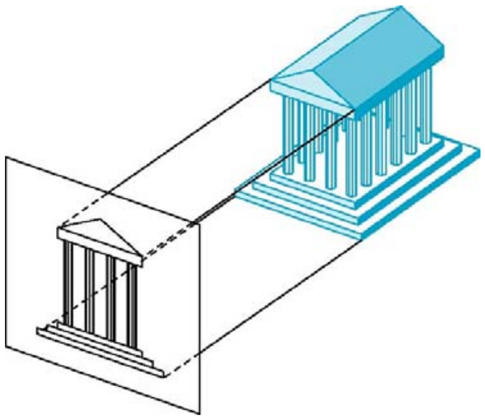


Perspective Viewing



Orthographic Projection

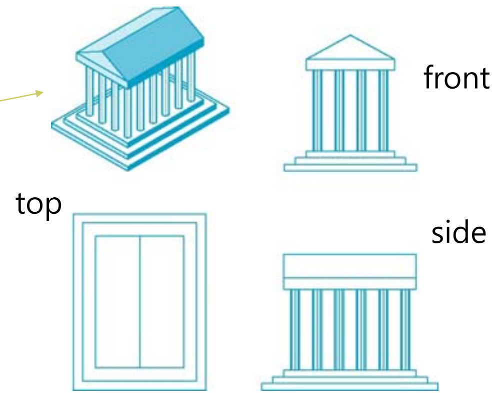
- In the orthographic projection, projectors are orthogonal to projection plane.



Multiview Orthographic Projection

- In the multiview orthographic projection, projection plane parallel to principal face.
- Usually form front, top, side views.

Isometric (not multiview orthographic view)



In CAD and architecture, we often display three multiviews plus isometric

Multiview Orthographic Projection Advantages and Disadvantages

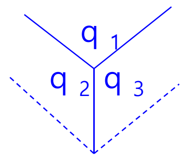
- Preserves both distances and angles
  - Shapes preserved
  - Can be used for measurements
    - Building plans
    - Manuals
- Cannot see what object really looks like because many surfaces hidden from view
  - Often we add the isometric

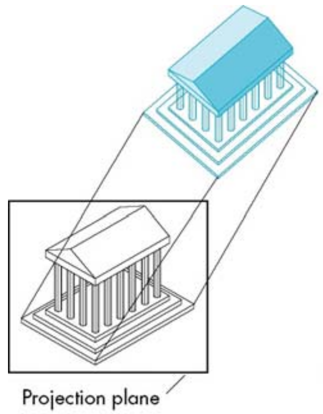
Axonometric Projections

- Axonometric projections allow projection plane to move relative to object.

classify by how many angles of a corner of a projected cube are the same

none: trimetric
two: dimetric
three: isometric





Construction of an Axonometric Projection

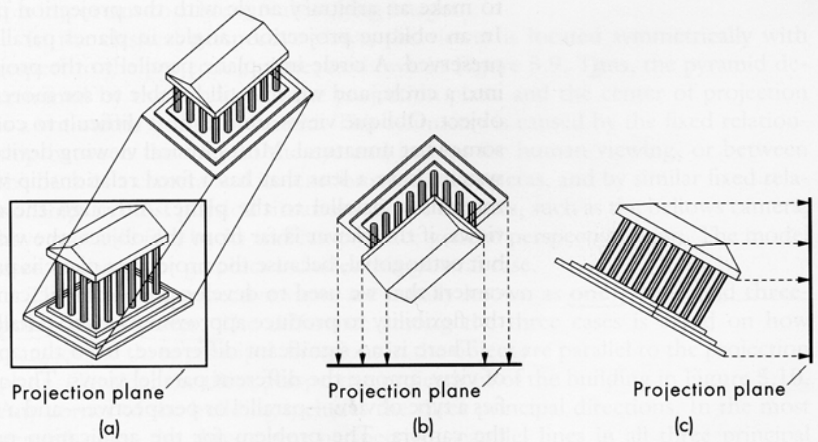
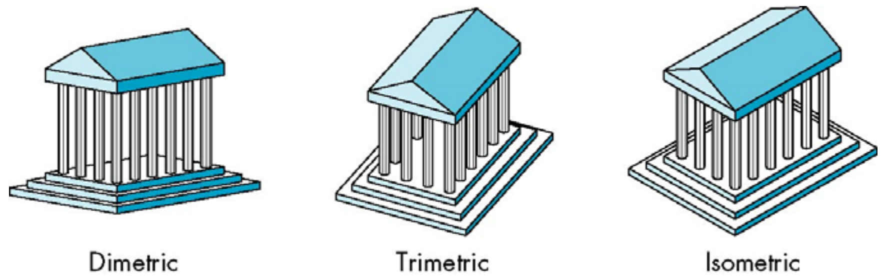


Figure 5.6 (a) Construction of an axonometric projection. (b) Top view. (c) Side view

Types of Axonometric Projections

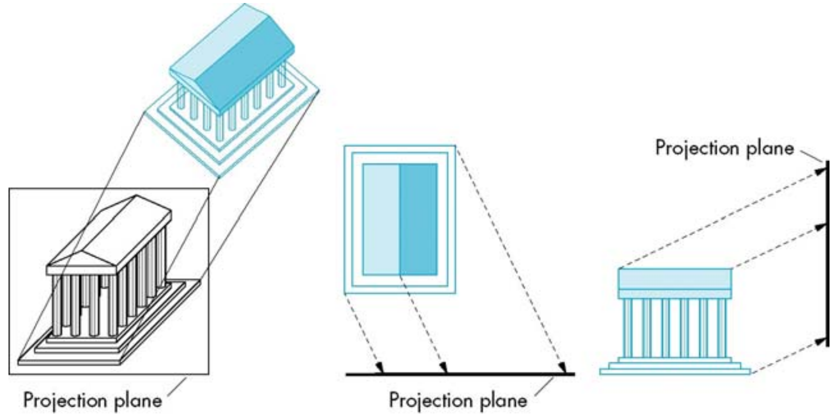


Axonometric Projections
Advantages and Disadvantages

- Lines are scaled (foreshortened) but can find scaling factors
- Lines preserved but angles are not
  - Projection of a circle in a plane not parallel to the projection plane is an ellipse
- Can see three principal faces of a box-like object
- Some optical illusions possible
  - Parallel lines appear to diverge
- Does not look real because far objects are scaled the same as near objects
- Used in CAD applications

Oblique Projection

- Arbitrary relationship between projectors and projection plane



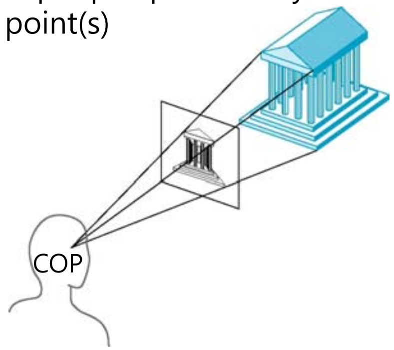
경사 투영의 평면도 & 측면도

Oblique Projection
Advantages and Disadvantages

- Can pick the angles to emphasize a particular face
  - Architecture: plan oblique, elevation oblique
- Angles in faces parallel to projection plane are preserved while we can still see "around" side
- In physical world, cannot create with simple camera; possible with bellows camera or special lens (architectural)

Perspective Projection

- Parallel lines (not parallel to the projection plan) on the object converge at a single point in the projection (the *vanishing point*)
- Drawing simple perspectives by hand uses these vanishing point(s)



1-,2-,3-Point Perspective

- Three-point perspectives – no principal face parallel to projection plane, 3 vanishing points.
- Two-point perspectives – on principal direction parallel to projection plane, 2 vanishing points.
- One-point perspective – one principal face parallel to projection plane, 1 vanishing point.

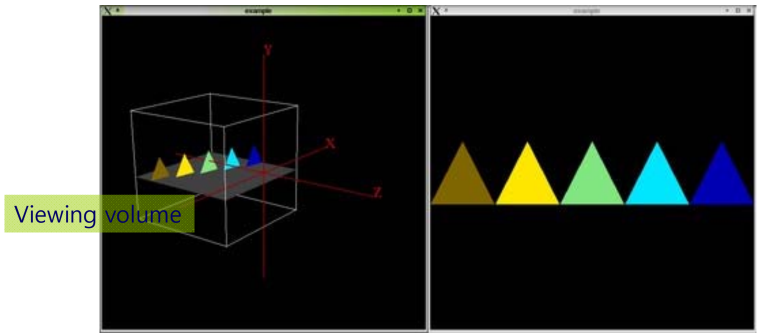
3-point perspective 2-point perspective 1-point perspective

Perspective Projections Advantages and Disadvantages

- Objects further from viewer are projected smaller than the same sized objects closer to the viewer (*diminution*)
  - Looks realistic
- Equal distances along a line are not projected into equal distances (*nonuniform foreshortening*)
- Angles preserved only in planes parallel to the projection plane
- More difficult to construct by hand than parallel projections (but not more difficult by computer)

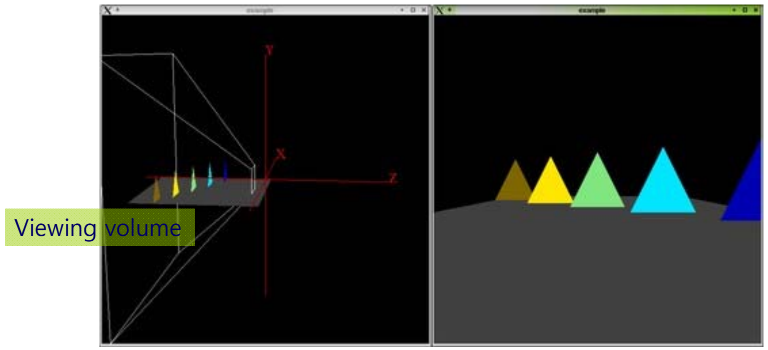
Orthographic Projection

- Orthographic projection projects the rectilinear box viewing volume onto the screen.
- The size of the object does not change with distance.



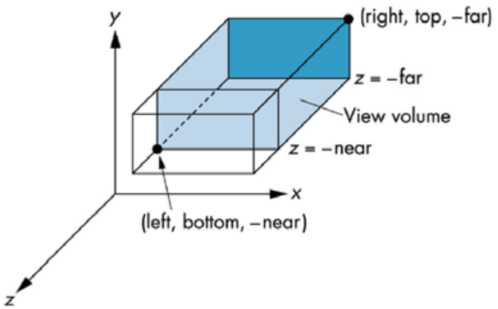
Perspective Projection

- Perspective projection projects the *frustum (i.e., truncated pyramid)* viewing space onto the screen.
- Near objects appear larger, and object far away appear smaller.



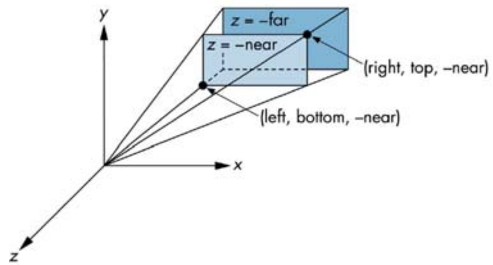
OpenGL Orthographic Projection

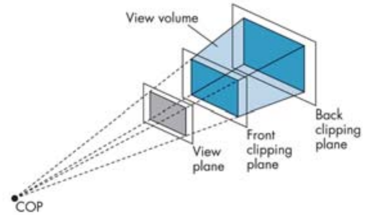
- **glm::ortho(left, right, bottom, top, near, far)**
  - The parameters of this function are the same as those of glm::frustum.
  - The viewing volume is rectilinear box.
  - Near and far take only positive numbers. It is used by changing it to a negative number inside.



OpenGL Perspective Projection

- In OpenGL perspective projection, the camera is positioned at the origin and is looking at the –Z-axis.
- **glm::frustum(left, right, bottom, top, near, far)**
  - The distance between near and far must be positive and is measured as the distance from the CPO to the near/far plane.
  - The viewing volume is frustum (i.e., truncated pyramid).

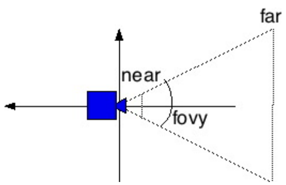


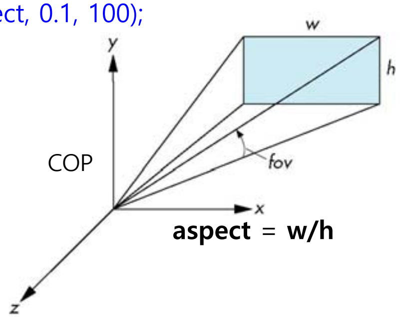


OpenGL Perspective Projection

- **glm::perspective(fovy, aspect, near, far)**
  - fovy – angle of field of view in Y-axis direction
  - aspect – the aspect ratio (width divided by height)
  - near – near clipping plane
  - far – far clipping plane

Projection = glm::perspective(45, aspect, 0.1, 100);



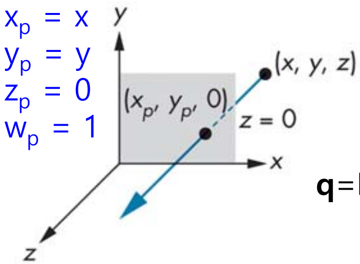


Orthographic Projection

- Orthographic projection
  - Special case of parallel projection in which the projector is orthogonal to the projection plane.
  - The focal length is infinite.

Orthographic projection

$x_p = x$
$y_p = y$
$z_p = 0$
$w_p = 1$

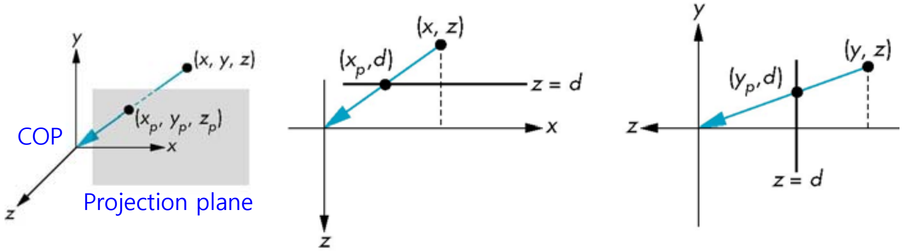


$$\mathbf{M}_{ortho} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$q = Mp \quad \mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \overset{\mathbf{M}_{ortho}}{\Rightarrow} \mathbf{q} = \begin{bmatrix} x \\ y \\ 0 \\ 1 \end{bmatrix}$$

Perspective Projection

- Perspective projection
  - Center of projection is located at the origin
  - Projection plane $z_p = d$



$$x_p = \frac{x}{z/d} \qquad y_p = \frac{y}{z/d} \qquad z_p = d$$

Perspective Projection

Perspective projection

$$x_p = \frac{x}{z/d}$$

$$y_p = \frac{y}{z/d}$$

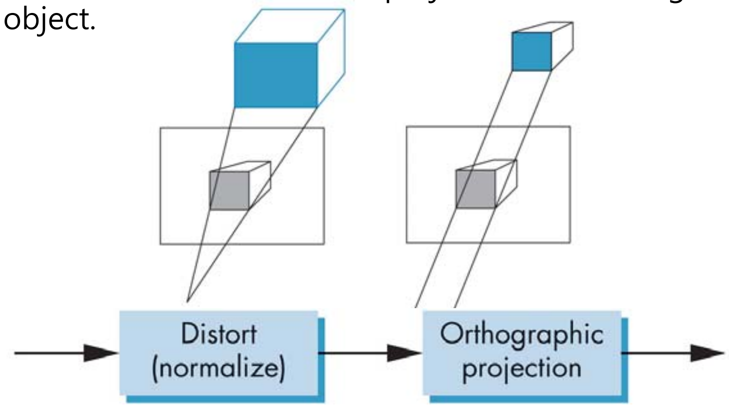
$$z_p = d = \frac{z}{z/d}$$

$$\mathbf{M}_{pers} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix}$$

$$\mathbf{q} = \mathbf{M}\mathbf{p} \qquad \mathbf{p} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \overset{\mathbf{M}_{pers}}{\Rightarrow} \mathbf{q} = \begin{bmatrix} x \\ y \\ z \\ z/d \end{bmatrix}$$

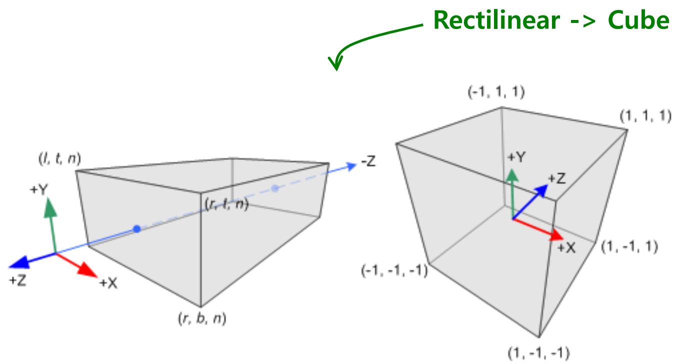
Projection Normalization

- Projection normalization converts all projections into orthogonal projections by distorting the objects such that the orthogonal projection of the distorted object is the same as the desired projection of the original object.



Orthogonal Projection Matrix

- Orthogonal projection maps a rectilinear view volume to Canonical view volume.



Orthogonal Projection Matrix

- Translate the center of viewing volume to the origin

$$T\left(-\frac{(right+left)}{2}\quad -\frac{(top+bottom)}{2}\quad -\frac{(-far+(-near))}{2}\right)$$

- Scale the viewing volume so that its length is 2x2x2

$$S\left(\frac{2}{(right-left)}\quad \frac{2}{(top-bottom)}\quad \frac{2}{(-far-(-near))}\right)$$

- P=ST= $\begin{pmatrix} \frac{2}{(right-left)} & 0 & 0 & -\frac{(right+left)}{(right-left)} \\ 0 & \frac{2}{(top-bottom)} & 0 & -\frac{(top+bottom)}{(top-bottom)} \\ 0 & 0 & -\frac{2}{(far-near)} & -\frac{(far+near)}{(far-near)} \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Orthogonal Projection Matrix

Ortho=ST

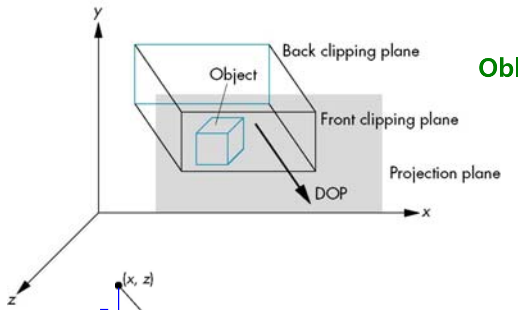
$$= \begin{pmatrix} \frac{2}{(right-left)} & 0 & 0 & 0 \\ 0 & \frac{2}{(top-bottom)} & 0 & 0 \\ 0 & 0 & -\frac{2}{(far-near)} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & -\frac{(right+left)}{2} \\ 0 & 1 & 0 & -\frac{(top+bottom)}{2} \\ 0 & 0 & 1 & \frac{(far+near)}{2} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{2}{(right-left)} & 0 & 0 & -\frac{(right+left)}{(right-left)} \\ 0 & \frac{2}{(top-bottom)} & 0 & -\frac{(top+bottom)}{(top-bottom)} \\ 0 & 0 & -\frac{2}{(far-near)} & -\frac{(far+near)}{(far-near)} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

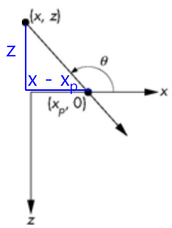
Orthogonal Projection Matrix

```
template <typename T> GLM_FUNC_QUALIFIER tmat4x4<T, defaultp> ortho
(T left, T right, T bottom, T top, T zNear, T zFar ) {

    tmat4x4<T, defaultp> Result(1);

    Result[0][0] = static_cast<T>(2) / (right - left);
    Result[1][1] = static_cast<T>(2) / (top - bottom);
    Result[2][2] = - static_cast<T>(2) / (zFar - zNear);

    Result[3][0] = - (right + left) / (right - left);
    Result[3][1] = - (top + bottom) / (top - bottom);
    Result[3][2] = - (zFar + zNear) / (zFar - zNear);

    return Result;
}
```

Oblique Projection Matrix



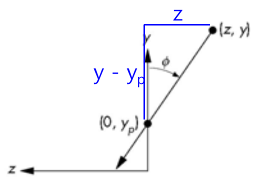
Oblique -> Orthogonal



top view

$$\tan\theta = \frac{z}{x-x_p}$$

$$x_p = x - z\cot\theta$$



side view

$$\tan\phi = \frac{z}{y-y_p},$$

$$y_p = y - z\cot\phi$$

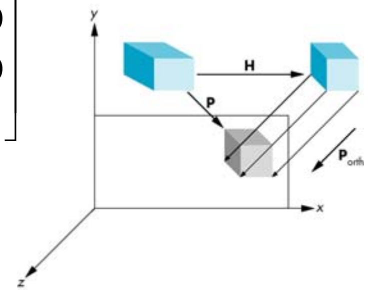
Oblique Projection Matrix

- *xy* shear (*z* values unchanged)

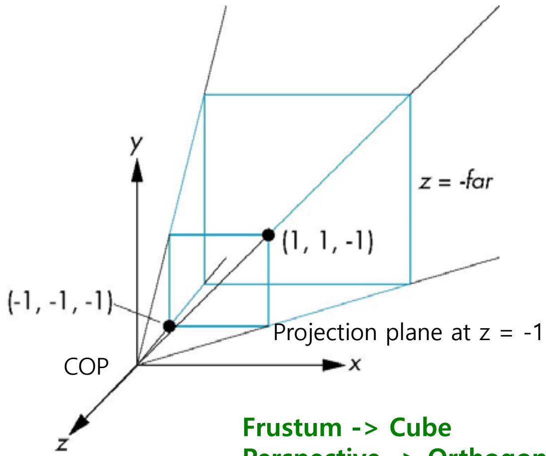
$$\mathbf{H}(\theta,\phi) = \begin{bmatrix} 1 & 0 & -\cot\theta & 0 \\ 0 & 1 & -\cot\varphi & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $\mathbf{P} = \mathbf{M}_{ortho}\ \mathbf{H}(\theta,\phi)$

- General case: $\mathbf{P} = \mathbf{M}_{ortho}\ \boxed{\mathbf{ST}}\ \mathbf{H}(\theta,\phi)$



Perspective Projection Matrix



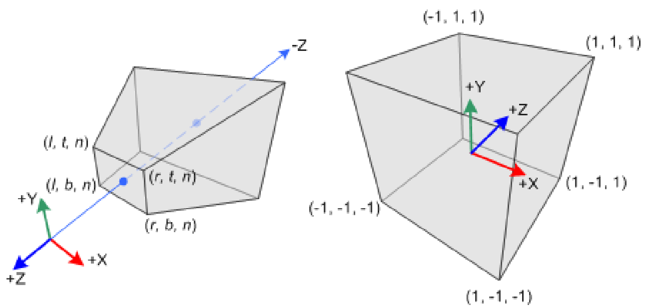
Frustum -> Cube
Perspective -> Orthogonal

Perspective Projection Matrix

- Perspective projection maps a frustum view volume to Canonical view volume.

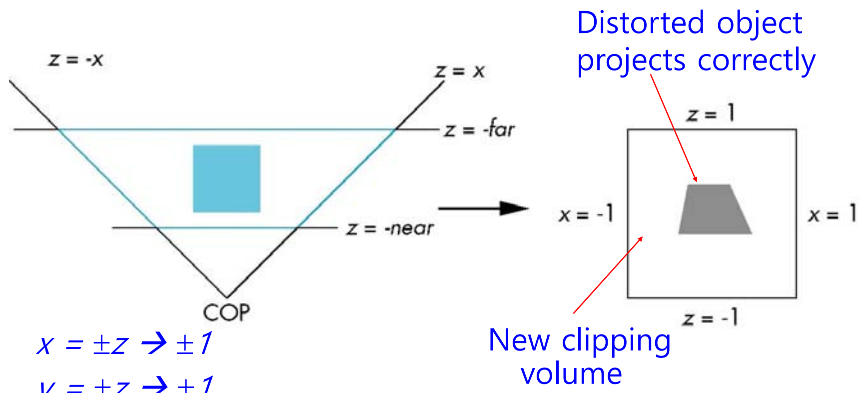
  [l, r] => [-1, 1], [b, t] => [-1, 1], [-n, -f] => [-1, 1]

  [n, f] => [1, -1]



Perspective Projection Matrix

- Perspective normalization



$x = \pm z \rightarrow \pm 1$

$y = \pm z \rightarrow \pm 1$

$z = near/far \rightarrow \pm 1$

Perspective Projection Matrix

- Perspective normalization converts perspective projection to orthogonal projection.
  - Perspective projection matrix with the projection plane as z = -1, and the center of projection as the origin, M

$$\mathbf{M}_{pers} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

  - The field of view is fixed at 90 degrees by making the side of the viewing volume as 45 degree.

$$x = \pm z$$

$$y = \pm z$$

Perspective Projection Matrix

- N matrix:

$$\mathbf{N} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

- p′=Np:

$$x' = x, \;\; y' = y, \;\; z' = \alpha z + \beta, \;\; w' = -z$$

- Perspective division, p′->p″:

$$\Rightarrow x'' = -\frac{x}{z}, \;\; y'' = -\frac{y}{z}, \;\; z'' = \frac{\alpha z + \beta}{-z}$$

Perspective Projection Matrix

- If x = ± z, x″ = ± 1
- If y = ± z, y″ = ± 1
- If far plane z = -far, $z'' = \frac{\alpha(-far) + \beta}{far} = 1$

  If near plane z = -near, $z'' = \frac{\alpha(-near) + \beta}{near} = -1$

- To become z″ -> ± 1, select α and β: (-near, -1) & (-far, 1)

$$\alpha = -\frac{far + near}{far - near}$$

$$\beta = -\frac{2\, far \;\; near}{far - near}$$

$$\alpha(-far) + \beta = far \;\&\; \alpha(-near) + \beta = -near$$

$$\beta = -near + \alpha near$$

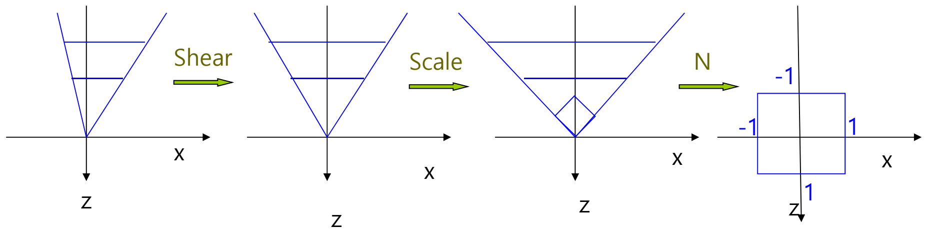
$$\alpha(-far) + (-near + \alpha near) = far$$

$$\alpha(near - far) = near + far$$

$$\alpha = \frac{near + far}{near - far} = -\frac{far + near}{far - near}$$

Perspective Projection Matrix

- glm::frustum(left, right, bottom, top, near, far)



Perspective Projection

- Shear $H(\cot\theta,\cot\phi)=H\left(\frac{(right+left)}{2near} \quad \frac{(top+bottom)}{2near}\right)$
- Then, $x=\pm\frac{right-left}{2near}$ $y=\pm\frac{top-bottom}{2near}$ $z=-near, z=-far$
- Scale $S\left(\frac{2near}{(right-left)} \quad \frac{2near}{(top-bottom)} \quad 1\right)$
- Then, $x=\pm z$ $y=\pm z$ $z=-near, z=-far$
- Normalize

$$\mathbf{N} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \alpha & \beta \\ 0 & 0 & -1 & 0 \end{bmatrix} \quad \alpha = -\frac{far + near}{far - near} \quad \beta = -\frac{2far\ near}{far - near}$$

Perspective Projection Matrix

Frustum=NSH

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\frac{(far+near)}{(far-near)} & -\frac{2farnear}{(far-near)} \\ 0 & 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} \frac{2near}{(right-left)} & 0 & 0 & 0 \\ 0 & \frac{2near}{(top-bottom)} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & \frac{(right+left)}{2near} & 0 \\ 0 & 1 & \frac{(top+bottom)}{2near} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

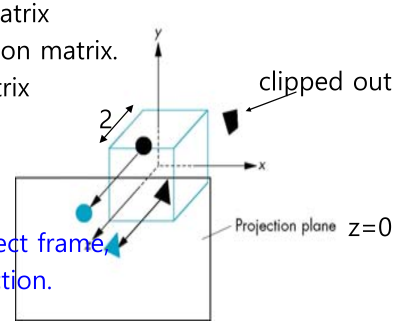
$$= \begin{pmatrix} \frac{2near}{(right-left)} & 0 & \frac{(right+left)}{(right-left)} & 0 \\ 0 & \frac{2near}{(top-bottom)} & \frac{(top+bottom)}{(top-bottom)} & 0 \\ 0 & 0 & -\frac{(far+near)}{(far-near)} & -\frac{2farnear}{(far-near)} \\ 0 & 0 & -1 & 0 \end{pmatrix}$$

Perspective Projection Matrix

```
template <typename T>
 GLM_FUNC_QUALIFIER tmat4x4<T, defaultp> frustum
 (T left, T right, T bottom, T top, T nearVal, T farVal) {

   tmat4x4<T, defaultp> Result(0);

   Result[0][0] = (static_cast<T>(2) * nearVal) / (right - left);
   Result[1][1] = (static_cast<T>(2) * nearVal) / (top - bottom);

   Result[2][0] = (right + left) / (right - left);
   Result[2][1] = (top + bottom) / (top - bottom);

   Result[2][2] = -(farVal + nearVal) / (farVal - nearVal);
   Result[2][3] = static_cast<T>(-1);
   Result[3][2] = -(static_cast<T>(2) * farVal * nearVal) / (farVal - nearVal);

   return Result;
 }
```

Computer Viewing

- Viewing
  - Set the position and direction of the camera.
    - Model-view transformation matrix
  - Apply the projection transformation matrix.
    - Projection transformation matrix
  - Clipping
    - View volume
- Default camera in OpenGL
  - Is placed at the origin of the object frame,
  - Faces to the negative z-axis direction.
  - Set to orthogonal projection,
  - The viewing volume is a cube with a length of 2 on each side centered on the origin.
  - The default projection plane with z=0, the projection direction is parallel to the z-axis.



Positioning the Camera Frame

- Model-view transformation matrix
- View-orientation matrix using VRP, VPN, VUP
- Look-at function

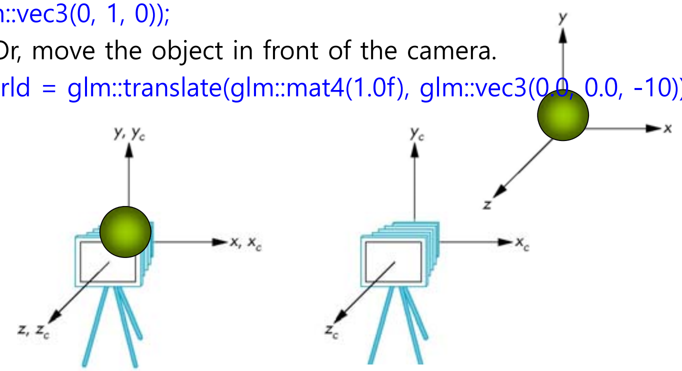
Positioning the Camera Frame

- Positioning the camera in OpenGL
  - Move the camera back from the origin

View = glm::lookAt(glm::vec3(0, 0, 10), glm::vec3(0, 0, 0), glm::vec3(0, 1, 0));

  - Or, move the object in front of the camera.

World = glm::translate(glm::mat4(1.0f), glm::vec3(0.0, 0.0, -10));



World frame = Camera frame

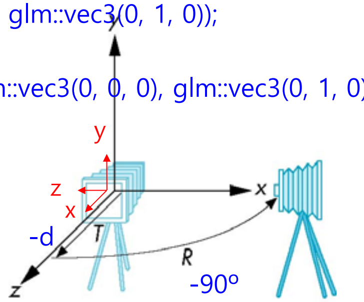
Moving the camera frame after translation by –d, d > 0

Positioning the Camera

- You can position the camera with successive rotation and translation.
- Viewing from the x-axis
  - R = rotate camera around y-axis
  - T = move the camera position away from the origin

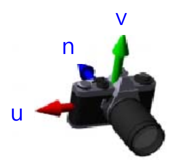
World = glm::translate(glm::mat4(1.0f), glm::vec3(0.0, 0.0, -10))
  * glm::rotate(glm::mat4(1.0f), -90, glm::vec3(0, 1, 0));

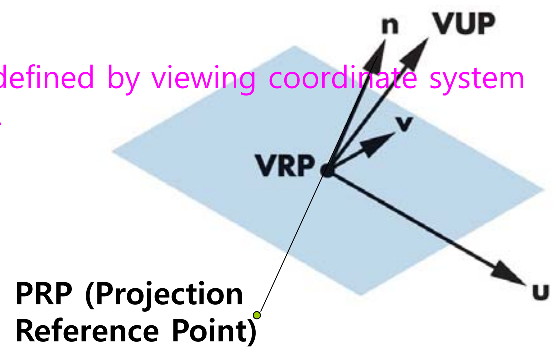
View = glm::lookAt(glm::vec3(10, 0, 0), glm::vec3(0, 0, 0), glm::vec3(0, 1, 0));



Camera Frame

- View reference point (VRP)
- View plane normal (VPN) n = VRP - PRP
- View-up vector (VUP)
- Side vector u = VUP x n
- Up vector v = n x u
- u, v, n normalize
- Camera frame is defined by viewing coordinate system (u'-v'-n') and VRP.





Camera Frame

- View-orientation matrix, M

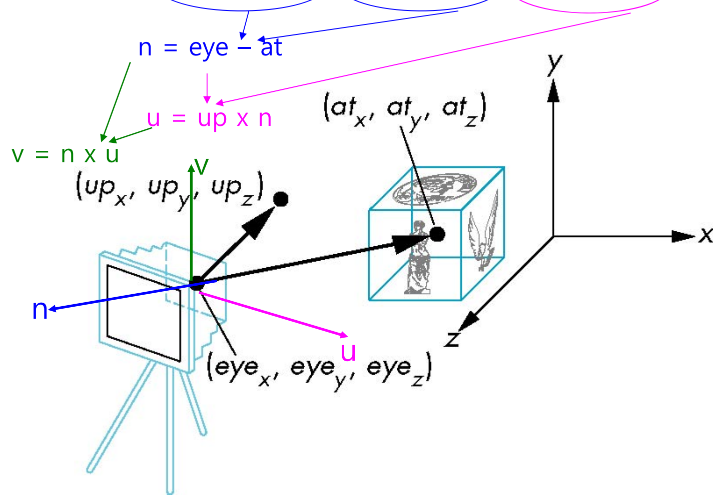
$$\mathbf{M} = \begin{bmatrix} u'_x & v'_x & n'_x & 0 \\ u'_y & v'_y & n'_y & 0 \\ u'_z & v'_z & n'_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Rotation matrix, $M^{-1} = M^T = R$
- Camera position in World frame: V = RT

$$\begin{bmatrix} u'_x & u'_y & u'_z & 0 \\ v'_x & v'_y & v'_z & 0 \\ n'_x & n'_y & n'_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & -e_x \\ 0 & 1 & 0 & -e_y \\ 0 & 0 & 1 & -e_z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} u'_x & u'_y & u'_z & -e \bullet u' \\ v'_x & v'_y & v'_z & -e \bullet v' \\ n'_x & n'_y & n'_z & -e \bullet n' \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

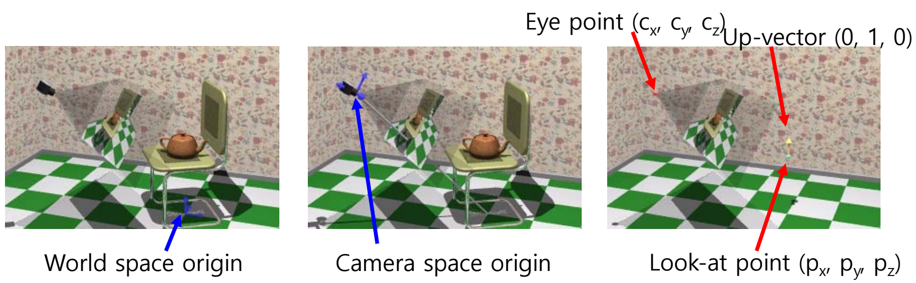
lookAt

- glm::lookAt(vec3 & eye, vec3 & at, vec3 & up)



lookAt

- *Eye Point* : camera origin (in World Coordinate System)
- *Look-At* : the position where the camera is looking at (the center of the camera image)
- *Up-Vector* : the camera up vector (in World Coordinate System)

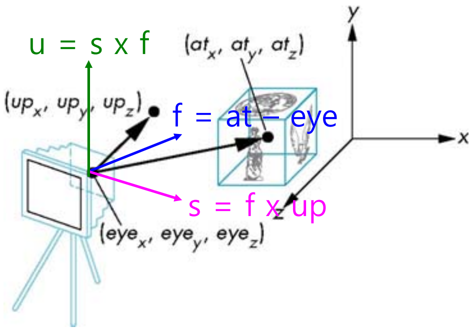


gluLookAt

```
void gluLookAt(GLdouble ex, GLdouble ey, GLdouble ez, GLdouble ax, GLdouble ay, GLdouble az,
    GLdouble ux, GLdouble uy, GLdouble uz) {
    GLdouble M[16]; GLdouble u[3], v[3], n[3]; GLdouble mag;

    n[0] = ex – ax; n[1] = ey – ay; n[2] = ez – az;          // n (camera frame Z)
    mag = sqrt(n[0]*n[0] + n[1]*n[1] + n[2]*n[2]);
    if (mag) { n[0] /= mag; n[1] /= mag; n[2] /= mag; }

    v[0] = ux; v[1] = uy; v[2] = uz;                         // u (camera frame X)
    u[0] = v[1]*n[2] – v[2]*n[1]; u[1] = -v[0]*n[2] + v[2]*n[0]; u[2] = v[0]*n[1] - v[1]*n[0];
    mag = sqrt(u[0]*u[0] + u[1]*u[1] + u[2]*u[2]);
    if (mag) { u[0] /= mag; u[1] /= mag; u[2] /= mag; }

    v[0] = n[1]*u[2] – n[2]*u[1]; v[1] = -n[0]*u[2] + n[2]*u[0]; v[2] = n[0]*u[1] - n[1]*u[0];  // v (camera
    frame Y)
    mag = sqrt(v[0]*v[0] + v[1]*v[1] + v[2]*v[2]);
    if (mag) { v[0] /= mag; v[1] /= mag; v[2] /= mag; }

    M[0] = u[0];  M[4] = u[1];  M[8]  = u[2];   M[12] = 0.0;     // R
    M[1] = v[0];  M[5] = v[1];  M[9]  = v[2];   M[13] = 0.0;
    M[2] = n[0];  M[6] = n[1];  M[10] = n[2];   M[14] = 0.0;
    M[3] = 0.0;   M[7] = 0.0;   M[11] = 0.0;    M[15] = 1.0;
    glMultMatrix(M);

    glTranslated(-ex, -ey, -ez);                               // RT
}
```

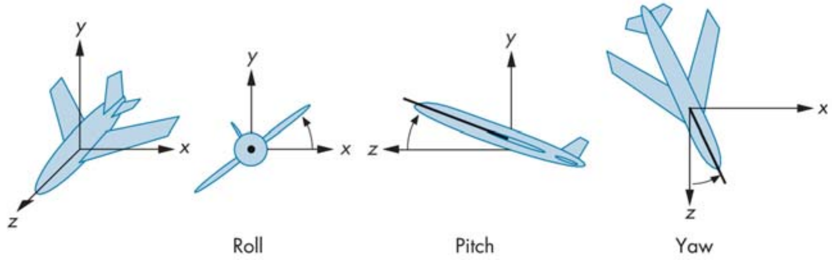
glm::lookAt Matrix

```
template <typename T, precision P>
 GLM_FUNC_QUALIFIER tmat4x4<T, P> lookAtRH
 (tvec3<T, P> const & eye, tvec3<T, P> const & center, tvec3<T, P> const & up) {
    tvec3<T, P> const f(normalize(center - eye));
    tvec3<T, P> const s(normalize(cross(f, up)));
    tvec3<T, P> const u(cross(s, f));
    tmat4x4<T, P> Result(1);
    Result[0][0] = s.x;
    Result[1][0] = s.y;
    Result[2][0] = s.z;
    Result[0][1] = u.x;
    Result[1][1] = u.y;
    Result[2][1] = u.z;
    Result[0][2] = -f.x;
    Result[1][2] = -f.y;
    Result[2][2] = -f.z;
    Result[3][0] = -dot(s, eye);
    Result[3][1] = -dot(u, eye);
    Result[3][2] = dot(f, eye);
    return Result;
}
```



Yaw, Pitch, Roll

- Yaw – Y-axis rotation
- Pitch – X-axis rotation
- Roll – Z-axis rotation



Elevation and Azimuth

- Azimuth – X-axis rotation (-180 ~ 180)
- Elevation – Y-axis rotation (-90 ~ 90)
- Twist angle – Z-axis rotation (-180 ~ 180)

Spherical Polar Coordinates System