

XNA Mathematics Class

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XNA Math & D3DXMath

XNAMath Type	D3DXMath Type
HALF	D3DXFLOAT16
XMVECTOR2	D3DXVECTOR2
XMVECTOR2_16F	D3DXVECTOR2_16F
XMVECTOR3	D3DXVECTOR3
XMVECTOR4	D3DXVECTOR4
XMVECTOR4_16F	D3DXVECTOR4_16F
XMMATRIX (or XMVECTOR4x4A)	D3DXMATRIXA16
XMVECTOR (or XMVECTOR4)	D3DXQUATERNION D3DXPLANE D3DXCOLOR

XNA Math & D3DXMath

XNAMath Macro	D3DXMath Macro
XM_PI	D3DX_PI
XM_1DIVPI	D3DX_1BYPI
XMConvertToRadians	D3DXToRadian
XMConvertToDegrees	D3DXToDegree

XNA Math & D3DXMath

XNAMath Function	D3DXMath Function
XMVector3Length	D3DXVec3Length
XMVector3Dot	D3DXVec3Dot
XMVector3Cross	D3DXVec3Cross
XMVectorAdd	D3DXVec2Add D3DXVec3Add
XMVectorSubtract	D3DXVec2Subtract D3DXVec3Subtract
XMVectorMin	D3DXVec2Minimize D3DXVec3Minimize
XMVectorMax	D3DXVec2Maximize D3DXVec3Maximize
.....

Vector – A Mathematical Definition

- Definition
 - A vector is a list of numbers
 - A vector is an array of numbers
- Vectors vs. Scalars
 - Scalar is not a vector quantity
 - Vector quantity: velocity, displacement
 - Scalar quantity: speed, distance
- Vector Dimension
 - Tell how many numbers the vector contains

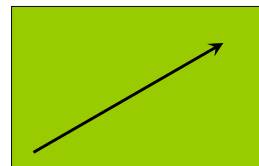
Vector – A Mathematical Definition

- Notation
 - Surround the list of numbers with square brackets, e.g.

$$[1 \quad 2 \quad 3] \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Vector – A Geometric Definition

- Geometrically speaking
 - A vector is a directed line segment that has **magnitude** and **direction**
 - The magnitude of a vector is the length of the vector
 - The direction of a vector describes which way the vector is pointing in space



A 2D vector

Position vs. Displacement

- Vectors do not have position
- **Only magnitude and direction**
- Example:
 - Displacement – e.g. Take three steps forward
 - Velocity – e.g. Traveling northeast at 50 MPH

Vector

□ XMVECTOR (xnamath.h)

- A portable type used to represent a vector of four 32-bit floating-point or integer components, each aligned optimally and mapped to a hardware vector register.
- Instances of XMVECTOR can be stored into an instance of XMFLOAT4 with XMStoreFloat4.

```
typedef struct _XMVECTOR {
    FLOAT x;
    FLOAT y;
    FLOAT z;
    FLOAT w;
} XMVECTOR;
```

Vector

□ XMVECTOR, XMVECTOR3 (xnamath.h)

```
typedef struct _XMVECTOR {
    FLOAT x;
    FLOAT y;
} XMVECTOR;
```

```
typedef struct _XMVECTOR3 {
    FLOAT x;
    FLOAT y;
    FLOAT z;
} XMVECTOR3;
```

Vector

□ XNA Framework Math provides Vector2, Vector3, Vector4 class

- **Vector2** (x, y)
 - [http://msdn.microsoft.com/en-us/library/microsoft.xna.framework.vector2_members\(v=xnagamestudio.30\).aspx](http://msdn.microsoft.com/en-us/library/microsoft.xna.framework.vector2_members(v=xnagamestudio.30).aspx)
- **Vector3** (x, y, z)
 - [http://msdn.microsoft.com/en-us/library/microsoft.xna.framework.vector3_members\(v=xnagamestudio.30\).aspx](http://msdn.microsoft.com/en-us/library/microsoft.xna.framework.vector3_members(v=xnagamestudio.30).aspx)
- **Vector4** (x, y, z, w)
 - [http://msdn.microsoft.com/en-us/library/microsoft.xna.framework.vector4_members\(v=xnagamestudio.30\).aspx](http://msdn.microsoft.com/en-us/library/microsoft.xna.framework.vector4_members(v=xnagamestudio.30).aspx)

3D Vector Operations

□ Equality

```
Vector3 u(1.0f, 0.0f, 1.0f);
Vector3 v(0.0f, 1.0f, 0.0f);
if (u == v) return true; // equal
if (u != v) return true; // not equal
```

□ Length(Magnitude)

```
Vector3 v(1.0f, 2.0f, 3.0f);
float magnitude = v.Length(); // =sqrt(14)
```

$$\|\vec{v}\|$$

□ Normalize

```
Vector3 v(1.0f, 2.0f, 3.0f);
v = v.Normalize(); // After this line executes, vector will be unit-length
```

$$\frac{\vec{v}}{\|\vec{v}\|}$$

3D Vector Operations

□ Addition: $u + v$

```
Vector3 u(2.0f, 0.0f, 1.0f);
Vector3 v(0.0f, -1.0f, 5.0f);
Vector3 sum = u + v; // (2.0+0.0, 0.0-1.0, 1.0+5.0) = (2.0, -1.0, 6.0)
```

□ Subtraction: $u - v$

```
Vector3 u(2.0f, 0.0f, 1.0f);
Vector3 v(0.0f, -1.0f, 5.0f);
Vector3 diff = u - v; // (2.0-0.0, 0.0+1.0, 1.0-5.0) = (2.0, 1.0, -4.0)
```

□ Scalar multiplication: $u * k$

```
Vector3 u(2.0f, 0.0f, -1.0f);
Vector3 scaleVec = u * 10.0f; // (2.0, 0.0, -1.0) * 10.0 = (20.0, 0.0, -10.0)
```

3D Vector Operations

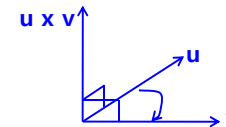
□ Dot product

```
Vector3 u(1.0f, -1.0f, 0.0f);
Vector3 v(3.0f, 2.0f, 1.0f);
float out = Vector3.Dot(u, v); // 1.0*3.0 + -1.0*2.0 + 0.0*1.0 = 1.0
```

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta = u_x v_x + u_y v_y + u_z v_z$$

□ Cross product

```
■  $u \times v = -(v \times u)$ 
Vector3 u(1.0f, -1.0f, 0.0f);
Vector3 v(3.0f, 2.0f, 1.0f);
Vector3 out = Vector3.Cross(u, v);
```



$$\vec{u} \times \vec{v} = (u_y v_z - u_z v_y, u_z v_x - u_x v_z, u_x v_y - u_y v_x)$$

3D Vector Operations

□ Zero

```
Vector3 v = Vector3.Zero; // v=Vector3(0, 0, 0)
```

□ Forward

```
Vector3 v = Vector3.Forward; // v=Vector3(0, 0, -1) RHS
```

□ Right

```
Vector3 v = Vector3.Right; // v=Vector3(1, 0, 0) RHS
```

□ Up

```
Vector3 v = Vector3.Up; // v=Vector3(0, 1, 0) RHS
```

□ UnitX

```
Vector3 v = Vector3.UnitX; // v=Vector3(1, 0, 0)
```

□ UnitY

```
Vector3 v = Vector3.UnitY; // v=Vector3(0, 1, 0)
```

□ UnitZ

```
Vector3 v = Vector3.UnitZ; // v=Vector3(0, 0, 1)
```

Matrix

□ A rectangular grid of numbers arranged into *rows* and *columns*.

□ Vector vs. Matrix

- Vector is an array of scalars
- Matrix is an array of vectors

□ Dimensions and Notation : $r \times c$ matrix

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} \left. \vphantom{\begin{pmatrix} m_{11} \\ m_{21} \\ m_{31} \end{pmatrix}} \right\} r(3) \text{ rows}$$

$$\underbrace{\hspace{10em}}_{c(3) \text{ columns}}$$

XMMatrix

□ XMMATRIX (xnamath.h)

- $v' = v_{1 \times 4} T_{4 \times 4}$ (not $T_{4 \times 4} v_{1 \times 4}$)

□ XMMATRIX

- `_ij` element = `ith` row number, `jth` column number

```
typedef struct _XMMATRIX {
    union {
        XMVECTOR r[4];
        struct {
            FLOAT _11; FLOAT _12; FLOAT _13; FLOAT _14;
            FLOAT _21; FLOAT _22; FLOAT _23; FLOAT _24;
            FLOAT _31; FLOAT _32; FLOAT _33; FLOAT _34;
            FLOAT _41; FLOAT _42; FLOAT _43; FLOAT _44; // translation(x,y,z)
        };
        FLOAT m[4][4];
    };
} XMMATRIX;
```

Matrix Operations

□ Matrix arithmetic operation: `==`, `+`, `-`, `*`, `/`

```
Matrix A(1,0,0,0,
         0,1,0,0,
         0,0,1,0,
         1,2,3,1); // A의 초기화
Matrix B(...); // B의 초기화
Matrix C = A * B; // C = AB
```

□ Matrix element access

```
Matrix m;
m.M11 = 5.0f; // _11 = 5.0f
m.M12 = 6.0f; // _12 = 5.0f
```

Matrix Operations

□ Transpose matrix M^T

```
Matrix A(...); // A 초기화
Matrix B;
B = Matrix.Transpose(A); // B = transpose(A)
```

□ Inverse matrix M^{-1}

```
Matrix A(...); // A 초기화
Matrix B;
B = Matrix.Invert(A); // B = inverse(A)
float det = A.Determinant(); // Determinant of matrix A
```

□ Identity matrix I

```
Matrix m = Matrix.Identity; // identity matrix
```

Matrix Operations

□ Translation matrix

```
Vector3 translationVector(1, 2, 3);
Matrix A = Matrix.CreateTranslation(translationVector);
```

□ Rotation matrix

```
Matrix ROTX = Matrix.CreateRotationX(MathHelper.ToRadians(60)); // 60°
Matrix ROTY = Matrix.CreateRotationY(MathHelper.Pi); // 180°
Matrix ROTZ = Matrix.CreateRotationZ(MathHelper.PiOver4); // 45°
```

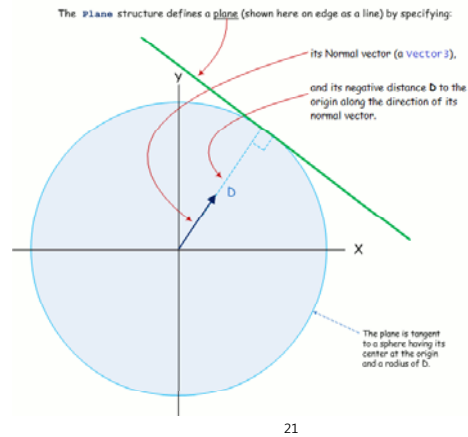
□ Scale matrix

```
Vector3 scaleVector(2, 2, 2);
Matrix S = Matrix.CreateScale(scaleVector);
Matrix SS = Matrix.CreateScale(2, 2, 2);
```

Plane

XMMLib Library Plane Functions

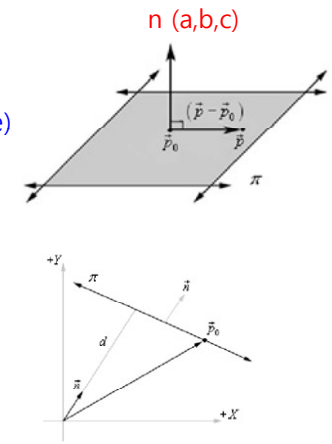
- XMPlaneDot
- XMPlaneDotCoord
- XMPlaneDotNormal
- XMPlaneEqual
- XMPlaneFromPointNormal
- XMPlaneFromPoints
- XMPlaneIntersectLine
- XMPlaneIntersectPlane
- XMPlaneIsInfinite
- XMPlaneIsNaN
- XMPlaneNearEqual
- XMPlaneNormalize
- XMPlaneNormalizeEst
- XMPlaneNotEqual
- XMPlaneTransform
- XMPlaneTransformStream



Plane

A plane is defined by a normal vector, n , and a point on the plane, p_0 :

- $ax + by + cz + d = 0$
- $n \cdot p + d = 0$
 $n = (a, b, c)$, $p = [n, d]$
 $d = -n \cdot p$ (the shortest signed distance)
- $n \cdot (p - p_0) = 0$
 p_0 is a point on the plane



Plane Construction

- Normal vector, n , and the signed distance, d :
 - Plane $p(a, b, c, d)$
- Normal vector, n , and a point on the plane, p_0 :
 - $d = -n \cdot p_0$
 Vector3 $n(a, b, c)$;
 Plane $p(n, d)$;
- Three points on the plane, p_0, p_1, p_2 :
 - $u = p_1 - p_0$; $v = p_2 - p_0$; $n = u \times v$; $d = -n \cdot p_0$
 Vector3 p_0, p_1, p_2 ;
 Plane $p(p_0, p_1, p_2)$;

Plane Normalization

- Normalizing a Plane
 - Normalize the plane normal vector. But, recall the normal vector influences the constant, d .
 - Therefore, if we normalize the normal vector, we must also recalculate d .

$$\frac{1}{\|n\|} (n, d) = \left(\frac{n}{\|n\|}, \frac{d}{\|n\|} \right)$$

Vector3 $n(a, b, c)$;
 Plane $p(n, d)$;
 p .**Normalize**();

Point and Plane Spatial Relationship

- Point and Plane Spatial Relationship
 - If $n \cdot p + d = 0$, then p is planar with the plane.
 - If $n \cdot p + d > 0$, then p is in front of the plane and in the plane's positive half space.
 - If $n \cdot p + d < 0$, then p is back of the plane and in the plane's negative half-space.
- PlaneDotCoord
 - Plane(a, b, c, d), Vector3(x, y, z), $a*x + b*y + c*z + d*1$

```
Plane p(0.0, 1.0, 0.0, 0.0); Vector3 v(3.0, 5.0, 2.0);
float x = p.DotCoord(v);
if (x approximately equal 0.0) // coplanar to the plane
if (x > 0) // in positive half-space
if (x < 0) // in negative half-space
```

```
-Approximately equal
const float EPSILON = 0.001f;
boolean Equals (float lhs, float rhs) { return fabs(lhs - rhs) < EPSILON ? true : false;}
```

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Relationship between Point and Plane

- XMPlaneDot
 - Plane(a, b, c, d), Vector4(x, y, z, w), $a*x + b*y + c*z + d*w$

```
XMVECTOR XMPlaneDot(XMVECTOR P, XMVECTOR V);
```
- XMPlaneDotCoord
 - Plane(a, b, c, d), Vector3(x, y, z), $a*x + b*y + c*z + d*1$

```
XMVECTOR XMPlaneDotCoord(XMVECTOR P, XMVECTOR V);
```
- XMPlaneDotNormal
 - Plane(a, b, c, d), Vector3(x, y, z), $a*x + b*y + c*z + d*0$

```
XMVECTOR XMPlaneDotNormal(XMVECTOR P, XMVECTOR V);
```

Plane Transformation

- Plane Transformation
 - Transform a normalized plane, $p=(n, d)$ by a matrix (or a quaternion): $p(T^{-1})^T$

```
// rendering simple water reflection
Vector3 planeWaterNormal = new Vector3(0, 1, 0);
Plane planeWater = new Plane(planeWaterNormal, 1);
planeWater.Normalize();
planeWater = Plane.Transform(planeWater, camera.View);
planeWater = Plane.Transform(planeWater, camera.Projection);
this.GraphicsDevice.ClipPlanes[0].Plane = planeWater;
this.GraphicsDevice.ClipPlanes[0].IsEnabled = true;
```

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Distance from a Point to a Plane

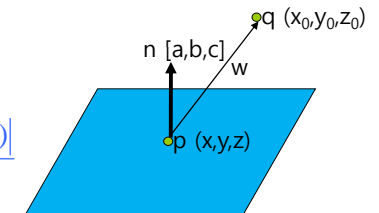
- **The shortest distance, D**, from a point, $q(x_0, y_0, z_0)$, to the plane, $P(n, d)$:
 - Point, q, lies in the plane if and only if $D=0$

$$w = [x_0 - x, y_0 - y, z_0 - z]$$

$$D = \frac{|n \cdot w|}{\|n\|}$$

$$= \frac{|a(x_0 - x) + b(y_0 - y) + c(z_0 - z)|}{\sqrt{a^2 + b^2 + c^2}}$$

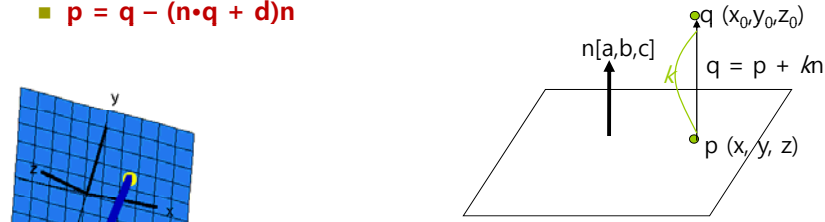
$$= \frac{ax_0 + by_0 + cz_0 + d}{\sqrt{a^2 + b^2 + c^2}}$$



```
Projecting w onto n : w|| = n * (w · n) / (n · n) & ||w|| = |w · n| / ||n||
```

Nearest Point on a Plane to a Particular Point

- Find a nearest point, $\mathbf{p}(x, y, z)$, on the plane, $P(n,d)$ to a particular point, $q(x_0, y_0, z_0)$:
 - $\mathbf{p} = \mathbf{q} - k\mathbf{n}$ (k =the shortest signed distance from a point, q , to the plane, P)
 - If \mathbf{n} is the unit vector, then $k = \mathbf{n} \cdot \mathbf{q} + d$
 - $\mathbf{p} = \mathbf{q} - (\mathbf{n} \cdot \mathbf{q} + d)\mathbf{n}$



$$\text{Distance}(q, \text{plane}) = \frac{ax_0 + by_0 + cz_0 + d}{\sqrt{a^2 + b^2 + c^2}}$$

where $q(x_0, y_0, z_0)$ and Plane $ax + by + cz + d = 0$

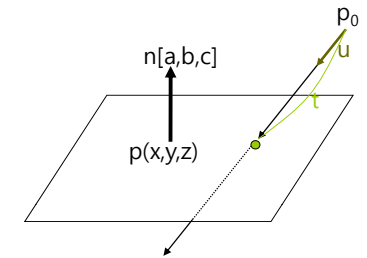
Intersection of Ray and Plane

- Ray: $\mathbf{p}(t) = \mathbf{p}_0 + t\mathbf{u}$
- Plane: $\mathbf{p} \cdot \mathbf{n} + d = 0$
- Ray/Plane Intersection:

$$(\mathbf{p}_0 + t\mathbf{u}) \cdot \mathbf{n} + d = 0$$

$$t\mathbf{u} \cdot \mathbf{n} = -d - \mathbf{p}_0 \cdot \mathbf{n}$$

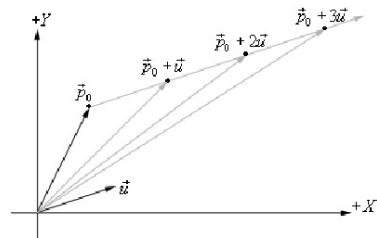
$$t = \frac{-(\mathbf{p}_0 \cdot \mathbf{n} + d)}{\mathbf{u} \cdot \mathbf{n}}$$



- No intersection, if a ray is parallel to the plane, i.e., $\mathbf{u} \cdot \mathbf{n} = 0$
- No intersection, if t is not in $[0, \infty)$, i.e., $t < 0$
- When intersected, $\mathbf{p}\left(\frac{-(\mathbf{p}_0 \cdot \mathbf{n} + d)}{\mathbf{u} \cdot \mathbf{n}}\right) = \mathbf{p}_0 + \frac{-(\mathbf{p}_0 \cdot \mathbf{n} + d)}{\mathbf{u} \cdot \mathbf{n}}\mathbf{u}$

Rays, Lines, and Line Segments

- Ray :
 - $\mathbf{p}(t) = \mathbf{p}_0 + t\mathbf{u}$ where \mathbf{p}_0 is the origin of the ray, $t \in [0, \infty)$
 - \mathbf{u} is a vector specifying the direction of the ray
- Line :
 - $\mathbf{p}(t) = \mathbf{p}_0 + t\mathbf{u}$ where $t \in [-\infty, \infty)$
- Line segment:
 - $\mathbf{p}(t) = \mathbf{p}_0 + t\mathbf{u}$ where $\mathbf{u} = \mathbf{p}_1 - \mathbf{p}_0$, $t \in [0, 1]$



Reference

- XNA Framework Math Overview (XNA Game Studio 4.0)
 - [http://msdn.microsoft.com/en-us/library/bb203910\(v=xnagamestudio.40\).aspx](http://msdn.microsoft.com/en-us/library/bb203910(v=xnagamestudio.40).aspx)
- XNA Math
 - [http://msdn.microsoft.com/en-us/library/windows/desktop/ee418725\(v=vs.85\).aspx](http://msdn.microsoft.com/en-us/library/windows/desktop/ee418725(v=vs.85).aspx)
- <http://www.math.umn.edu/~nykamp/m2374/readings/planedist/>