

Geometric Tests

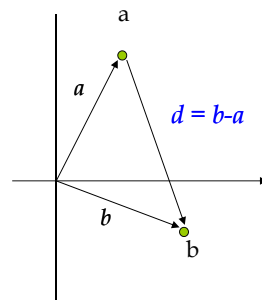
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 Spring 2014
 6/10/2014
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Overview

- Closest Point and Distance Test
 - Closest point on Line, Sphere, AABB, OBB, and Plane
 - Distance from Point to Point, Line, Sphere, AABB, OBB, and Plane
 - Distance from Sphere to Line, Sphere, AABB, OBB, and Plane
- Inside/Outside Test
 - Sphere, AABB, and OBB contains Point
 - Sphere contains Sphere
 - AABB contains AABB
- Intersection Test
 - Line-Line, Line-Sphere, Line-AABB, Line-Plane, Line-Triangle Intersection
 - Sphere-Sphere, Sphere-AABB, Sphere-Plane Intersection
 - AABB-AABB, AABB-Plane Intersection
 - OBB-OBB, OBB-Plane Intersection – not discussed today

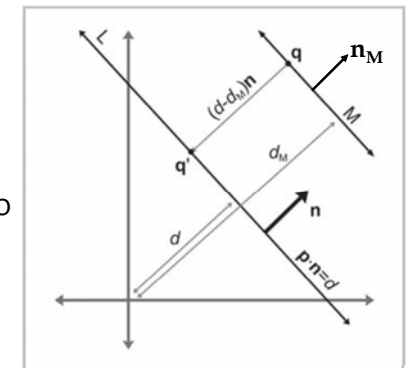
Distance

- Compute the distance between two points a and b
 - construct vector a
 - construct vector b
 - compute vector $d = b - a$
 - $\text{Distance}(a,b) = \|d\|$
 $= \|b - a\|$
 $= \sqrt{\text{dot}(d, d)}$



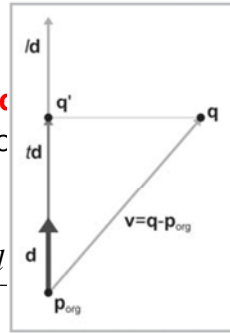
Closest Point on 2D Implicit Line

- Consider an infinite line L in 2D defined implicitly by all points p such that $p \cdot n = d$, where n is a unit vector.
- Find the point q' that is the closest point on L to q .
- Let n_M & d_M be the normal & d value of the line M .
- Since L and M are parallel,
 - $n_M = n$
 - $d_M = q \cdot n$
- The signed distance from M to L is $d - d_M = d - q \cdot n$
- $q' = q + (d - d_M)n$
 $= q + (d - q \cdot n)n$



Closest Point on Parametric Ray

- Consider the parametric ray R defined by: $\mathbf{p}(t) = \mathbf{p}_{org} + t\mathbf{d}$, where t varies from $0 \dots l$ (l is the length of R).
- For a given point \mathbf{q} , find the point \mathbf{q}' that is closest to \mathbf{q} .
- Let $\mathbf{v} = \mathbf{q} - \mathbf{p}_{org}$.
- Then, $t = \mathbf{v} \cdot \mathbf{d} = (\mathbf{q} - \mathbf{p}_{org}) \cdot \mathbf{d}$
- Then, $\mathbf{q}' = \mathbf{p}_{org} + t\mathbf{d} = \mathbf{p}_{org} + ((\mathbf{q} - \mathbf{p}_{org}) \cdot \mathbf{d})\mathbf{d}$
- If $t < 0$ or $t > l$, then $\mathbf{p}(t)$ is not in the portion contained by R .
 - If $t < 0$, \mathbf{q} will be the origin.
 - If $t > l$, \mathbf{q} will be endpoint.
- If \mathbf{d} is not a unit vector, there $= \frac{(\mathbf{q} - \mathbf{p}_{org}) \cdot \mathbf{d}}{\|\mathbf{d}\|}$



$$\text{Distance}(\mathbf{q}, \text{line}) = \sqrt{\|\mathbf{v}\|^2 - t^2}$$

Projecting \mathbf{v} onto \mathbf{d} : $v_{\parallel} = d \frac{\mathbf{v} \cdot \mathbf{d}}{\|\mathbf{d}\|^2}$ & $\|v_{\parallel}\| = \frac{\mathbf{v} \cdot \mathbf{d}}{\|\mathbf{d}\|}$

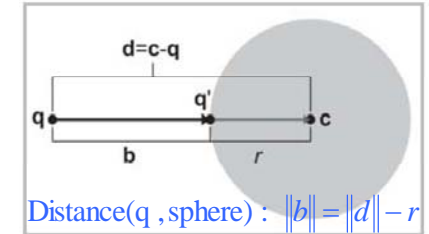
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Closest Point on Circle/Sphere

- Imagine a 2D point \mathbf{q} and a circle with center \mathbf{c} and radius r .
- Find \mathbf{q}' , which is the closest point on the circle to \mathbf{q} .
- Let \mathbf{d} be the vector from \mathbf{q} to \mathbf{c} . This vector intersects the circle at \mathbf{q}' .
- Let \mathbf{b} be the vector from \mathbf{q} to \mathbf{q}' .

$$\mathbf{b} = \frac{\|\mathbf{d}\| - r}{\|\mathbf{d}\|} \mathbf{d}$$

- Thus, $\mathbf{q}' = \mathbf{q} + \mathbf{b}$
- $= \mathbf{q} + \frac{\|\mathbf{d}\| - r}{\|\mathbf{d}\|} \mathbf{d}$



Distance(\mathbf{q} , sphere) : $\|\mathbf{b}\| = \|\mathbf{d}\| - r$

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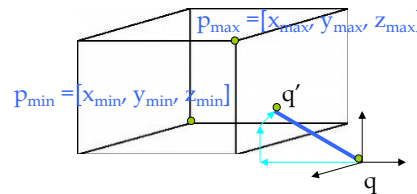
Closest Point on AABB

- Let AABB defined by the extreme points \mathbf{p}_{min} and \mathbf{p}_{max} .
- Compute \mathbf{q}' , the closest point in AABB to \mathbf{q} .
- This is done by "pushing" \mathbf{q} into AABB along each axis.

if $q[0] < \min[0]$: $q[0] = \min[0]$
 elif $q[0] > \max[0]$: $q[0] = \max[0]$

if $q[1] < \min[1]$: $q[1] = \min[1]$
 elif $q[1] > \max[1]$: $q[1] = \max[1]$

if $q[2] < \min[2]$: $q[2] = \min[2]$
 elif $q[2] > \max[2]$: $q[2] = \max[2]$



Distance(\mathbf{q} , AABB) : $\|\mathbf{q} - \mathbf{q}'\|$

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Closest Point on OBB

- Closest point on the OBB, \mathbf{q}'

$$x = \frac{B_x \cdot (\mathbf{q} - \mathbf{B}_{center})}{\|B_x\|}, \text{ if } x < -\|B_x\|, x = -\|B_x\| \text{ \& } dx = -\|B_x\| - x$$

$$\text{if } x > \|B_x\|, x = \|B_x\| \text{ \& } dx = x - \|B_x\|$$

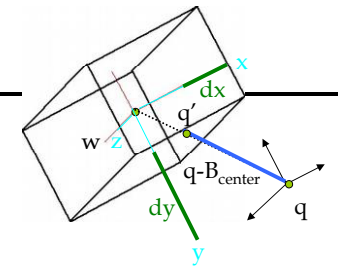
otherwise, $dx = 0$

$$y = \frac{B_y \cdot (\mathbf{q} - \mathbf{B}_{center})}{\|B_y\|}, \text{ if } y < -\|B_y\|, y = -\|B_y\|, dy = -\|B_y\| - y \text{ \& } \text{if } y > \|B_y\|, y = \|B_y\|, dy = y - \|B_y\| \text{ \& } dy = 0$$

$$z = \frac{B_z \cdot (\mathbf{q} - \mathbf{B}_{center})}{\|B_z\|}, \text{ if } z < -\|B_z\|, z = -\|B_z\|, dz = -\|B_z\| - z \text{ \& } \text{if } z > \|B_z\|, z = \|B_z\|, dz = z - \|B_z\| \text{ \& } dz = 0$$

- Distance from a point to the nearest point on the OBB

$$\text{Distance} = \sqrt{dx^2 + dy^2 + dz^2}$$



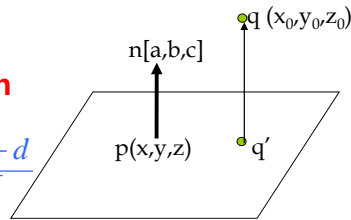
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Closest Point on Plane

- Consider a plane P defined by $\mathbf{p} \cdot \mathbf{n} + d = 0$
- Given a point \mathbf{q} , find the point \mathbf{q}' (the closest point on P to \mathbf{q}), which is the result of projecting \mathbf{q} onto P.
- Imagine a line containing the point \mathbf{q} on the plane P.
 - $\mathbf{q} = \mathbf{q}' + a\mathbf{n}$
 - $\mathbf{q} \cdot \mathbf{n} = \mathbf{q}' \cdot \mathbf{n} + (a\mathbf{n}) \cdot \mathbf{n}$
 - $\mathbf{q} \cdot \mathbf{n} = -d + a$
 - $a = \mathbf{q} \cdot \mathbf{n} + d$
- Therefore, $\mathbf{q}' = \mathbf{q} - (\mathbf{q} \cdot \mathbf{n} + d)\mathbf{n}$

$$\text{Distance}(\mathbf{q}, \text{plane}) = \frac{ax_0 + by_0 + cz_0 + d}{\sqrt{a^2 + b^2 + c^2}}$$

where $q(x_0, y_0, z_0)$ and Plane $ax + by + cz + d = 0$



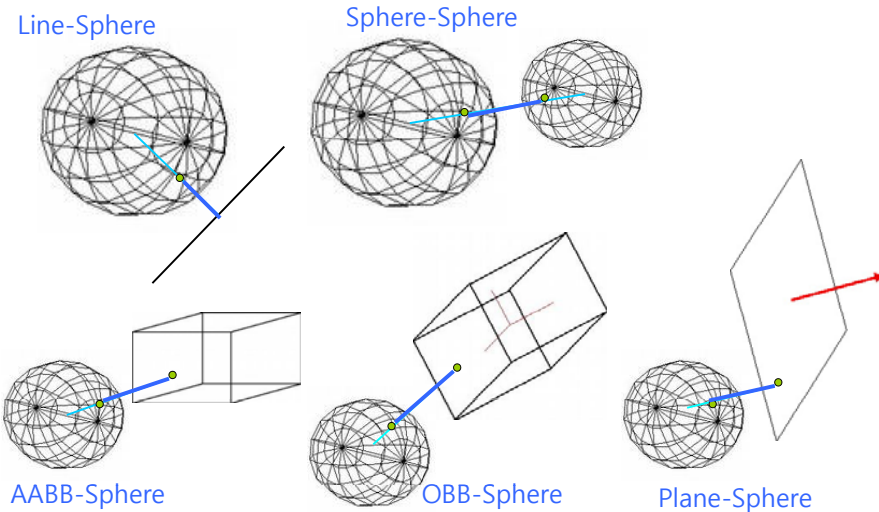
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Distance – Sphere to Others

- Distance from a line to the sphere
 - $\text{Distance}(\text{line}, s) = \text{Distance}(s.\text{center}, \text{line}) - s.\text{radius}$
- Distance from a sphere to the sphere
 - $\text{Distance}(\text{sphere}, s) = \text{Distance}(s.\text{center}, \text{sphere}) - s.\text{radius}$
- Distance from AABB to the sphere
 - $\text{Distance}(\text{aabb}, s) = \text{Distance}(s.\text{center}, \text{aabb}) - s.\text{radius}$
- Distance from OBB to the sphere
 - $\text{Distance}(\text{obb}, s) = \text{Distance}(s.\text{center}, \text{obb}) - s.\text{radius}$
- Distance from a plane to the sphere
 - $\text{Distance}(\text{plane}, s) = \text{Distance}(s.\text{center}, \text{plane}) - s.\text{radius}$

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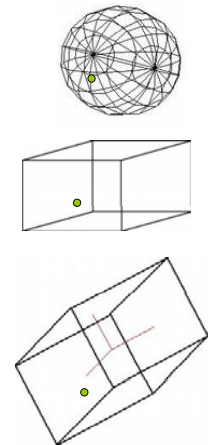
Distance – Sphere to Others



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Inside/Outside

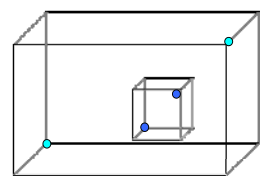
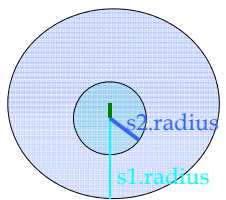
- Sphere contains a point
 - Point is inside Sphere
if $|p - c|^2 < r^2$
- AABB contains a point
 - Point is inside AABB
if $x_{\min} \leq p_x \leq x_{\max}$
and $y_{\min} \leq p_y \leq y_{\max}$
and $z_{\min} \leq p_z \leq z_{\max}$
- OBB contains a point
 - Point is inside OBB
if $(p - b_{\text{center}}) \cdot b_u \leq |b_u|^2$
and $(p - b_{\text{center}}) \cdot b_v \leq |b_v|^2$
and $(p - b_{\text{center}}) \cdot b_w \leq |b_w|^2$



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Inside/Outside

- Sphere contains sphere
 - Sphere is inside sphere
 - if $|c1 - c2|^2 < |r1 + r2|^2$
- AABB contains AABB
 - AABB is inside AABB
 - if $x1_{min} \leq x2_{min}$ and $x1_{max} \geq x2_{max}$
 - and $y1_{min} \leq y2_{min}$ and $y1_{max} \geq y2_{max}$
 - and $z1_{min} \leq z2_{min}$ and $z1_{max} \geq z2_{max}$



Intersection Tests

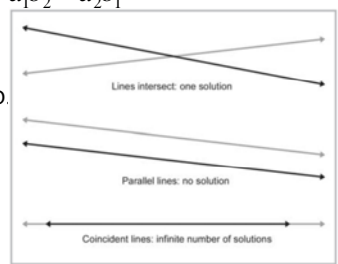
- These tests are designed to determine if two geometric primitives intersect and locate the intersection.
- The foundation for a collision detection system
 - To prevent objects from passing through each other
 - To make things appear to bounce off each other convincingly
- Two different types of intersection tests
 - A static test – checks two stationary primitives and detects if the two primitives intersect.
 - A dynamic test – checks two moving primitives and detects if and when two primitives intersect.
 - A time value (the value of the parameter t)
 - Easier to view the problem from the point of view of one of the primitives.

Intersection of Two Implicit Lines in 2D

- To find the intersection of two lines defined implicitly in 2D, use a straightforward matter of solving a system of linear equations.

$$\begin{aligned} a_1x + b_1y &= d_1 \\ a_2x + b_2y &= d_2 \end{aligned} \Rightarrow \begin{aligned} x &= \frac{b_2d_1 - b_1d_2}{a_1b_2 - a_2b_1} \\ y &= \frac{a_1d_2 - a_2d_1}{a_1b_2 - a_2b_1} \end{aligned}$$

- Three possibilities:
 - Intersect at one point – The denominators will be nonzero.
 - Parallel (i.e., no intersection) – The denominators are zero.
 - Coincident – The denominators are zero.

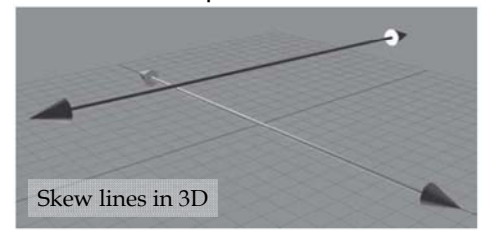


Intersection of Two Rays in 3D

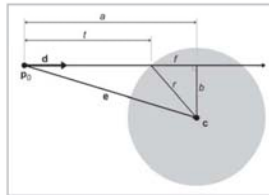
- Consider two rays in 3D defined parametrically by:
 - $p_1(t_1) = p_1 + t_1d_1$
 - $p_2(t_2) = p_2 + t_2d_2$
- If the rays lie in a plane, then there are three cases, similar to case of two lines.
- However, in 3D there is a fourth case where the rays are skew and do not share a common plane or intersect.

$$t_1 = \frac{(p_2 - p_1) \times d_2 \cdot (d_1 \times d_2)}{\|d_1 \times d_2\|^2}$$

$$t_2 = \frac{((p_2 - p_1) \times d_1) \cdot (d_1 \times d_2)}{\|d_1 \times d_2\|^2}$$



Intersection of Ray and Circle/Sphere



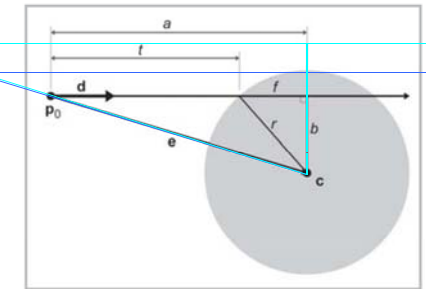
- We will consider the line \mathbf{e} that contains the ray and the center of circle.
- The circle/sphere is defined by its center \mathbf{c} and radius r .
- The ray defined by: $\mathbf{p}(t) = \mathbf{p}_0 + t\mathbf{d}$
- The value of t at the point of intersection is $t = a - f$.
- a is computed by $a = \mathbf{e} \cdot \mathbf{d}$, where $\mathbf{e} = \mathbf{c} - \mathbf{p}_0$.
- By the Pythagorean theorem, $r^2 + b^2 = e^2$.
- On the larger triangle, we can solve for b^2 . $a^2 + b^2 = e^2$.
- As a result, $r^2 = e^2 - a^2$,
- Finally, solving for t : $t = a - f = a - \sqrt{r^2 - e^2 + a^2}$
- Ray does not intersect the sphere if $r^2 - e^2 + a^2 < 0$

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Intersection of Ray and Circle/Sphere

```

 $\mathbf{e} = \mathbf{c} - \mathbf{p}_0;$ 
 $a = \text{dot}(\mathbf{e}, \mathbf{d});$ 
if ( $r^2 - \mathbf{e} \cdot \mathbf{e} + a^2 < 0$ ) {
    return None;
}
elseif ( $r^2 - \mathbf{e} \cdot \mathbf{e} + a^2 == 0$ ) {
     $t = a;$ 
    return  $\mathbf{p}_0 + t \cdot \mathbf{d};$ 
}
else {
     $f = \text{sqrt}(r^2 - \mathbf{e} \cdot \mathbf{e} + a^2);$ 
     $t_1 = a - f;$ 
     $t_2 = a + f;$ 
    return Ray( $\mathbf{p}_0 + t_1 \cdot \mathbf{d}$ ,  $\mathbf{p}_0 + t_2 \cdot \mathbf{d}$ );
}
    
```



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Intersection of Ray and AABB

- Computing the intersection of a ray and an AABB is an important calculation.
 - The result of this test is commonly used for trivial rejection on more complicated objects.
 - For example, ray-tracing against multiple triangle meshes,
 - First, ray trace against the AABBs of the meshes to trivially reject entire meshes at once.
 - Then, ray trace against each triangle not rejected.

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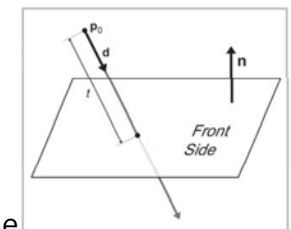
Intersection of Ray and Plane

- Let the ray be defined by $\mathbf{p}(t) = \mathbf{p}_0 + t\mathbf{d}$ and the plane be defined, by all points \mathbf{p} such that $\mathbf{p} \cdot \mathbf{n} + d = 0$.
- To solve for t at the point of intersection, assume an infinite ray for the moment.

$$(\mathbf{p}_0 + t\mathbf{d}) \cdot \mathbf{n} + d = 0$$

$$t\mathbf{d} \cdot \mathbf{n} = -d - \mathbf{p}_0 \cdot \mathbf{n}$$

$$t = \frac{-d - \mathbf{p}_0 \cdot \mathbf{n}}{\mathbf{d} \cdot \mathbf{n}}$$



- If the ray is parallel to the plane, then the denominator $\mathbf{d} \cdot \mathbf{n} = 0$ and there is no intersection.
- If the value of t is out of range, then the ray does not intersect the plane.

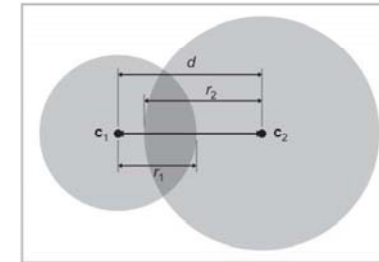
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Intersection of Ray and Triangle

- The ray-triangle intersection test is very important in graphics and computational geometry.
- To find the intersection of ray and triangle,
 - First, compute the point where the ray intersects the plane containing the triangle.
 - Then, test to see if the point is inside the triangle by computing the barycentric coordinates of the point.
- To make this test as fast as possible,
 - Detect and return a negative result (no collision) as soon as possible.
 - Defer expensive mathematical operations, such as division, as long as possible
 - Only detect collisions where the ray approaches the triangle from the front side
- Optimizing expensive calculations, such as precomputing the polygon normal, is one more significant strategy.

Intersection of Two Circles/Spheres

- We will consider two spheres defined by centers \mathbf{c}_1 and \mathbf{c}_2 and radii r_1 and r_2 , respectively.

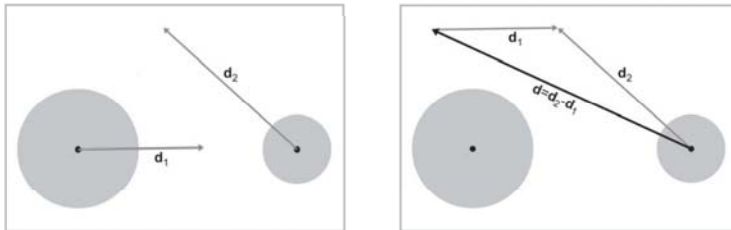


- Detection of the static intersection of two spheres
 - Let d be the distance between their centers \mathbf{c}_1 and \mathbf{c}_2 .
 - Then, the spheres intersect if $d < r_1 + r_2$.
 - In practice, we use $d^2 < (r_1 + r_2)^2$ for avoiding the square root.

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Intersection of Two Circles/Spheres

- Detection of the intersection of two moving spheres
- Assume two spheres with displacement vector \mathbf{d}_1 and \mathbf{d}_2 , respectively.

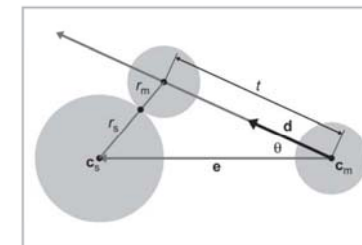


- Consider that sphere to be "stationary", while the other sphere is the "moving" sphere.
- Then, we can compute a single displacement vector \mathbf{d} .

$$\mathbf{d} = \mathbf{d}_2 - \mathbf{d}_1$$

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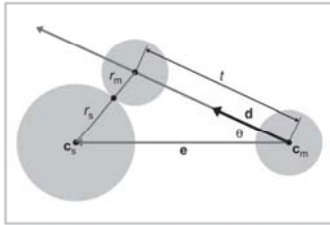
Intersection of Two Circles/Spheres



- Let the stationary sphere be defined by its center \mathbf{c}_s and radius r_s .
- We will normalize \mathbf{d} and vary t from 0 ... 1, where 1 is the distance traveled.
- The position of the center of the moving sphere at time t : $\mathbf{c}_m + t\mathbf{d}$.
- Let $\mathbf{e} = \mathbf{c}_s - \mathbf{c}_m$ and $r = r_m + r_s$.

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Intersection of Two Circles/Spheres



- $r^2 = t^2 + |e|^2 - 2t|e|\cos\theta$ (the law of cosines)
- $0 = t^2 - 2(e \cdot d)t + e \cdot e - r^2$ (the geometric interpretation of the dot product)

$$t = \frac{2(e \cdot d) \pm \sqrt{(-2(e \cdot d))^2 - 4(e \cdot e - r^2)}}{2} = e \cdot d \pm \sqrt{(e \cdot d)^2 + r^2 - e \cdot e}$$

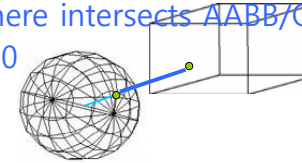
- → begin point of intersection
+ → cease point of intersection

- If $|e|^2 < r^2$, sphere are intersecting at $t=0$.

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Intersection of Sphere and AABB/OBB

- To detect the static intersection of a sphere and an AABB/OBB
 - Find the point on the AABB/OBB that is closest to the center of the sphere.
 - Compute the distance from this point to the center of the sphere.
 - Compare this distance with the radius.
 - The sphere intersects AABB/OBB if this distance is smaller than the radius.
- The sphere intersects AABB/OBB if $\text{Distance}(\text{sphere}, \text{box}) \leq 0$



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Intersection of Sphere and Plane

- To detect the static intersection of a sphere and a plane
- We simply compute the distance from the center of the sphere to the plane.

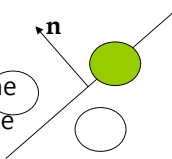
$$\text{Distance}(c, \text{plane}) = \frac{ac_x + bc_y + cc_z + d}{\sqrt{a^2 + b^2 + c^2}}$$

where $c(c_x, c_y, c_z)$ and Plane $ax + by + cz + d = 0$

- If this distance is less than the radius of the sphere, then the sphere intersects the plane.

- Three classifications

- The sphere is completely on the front of the plane
- The sphere is completely on the back of the plane
- The plane straddles the sphere



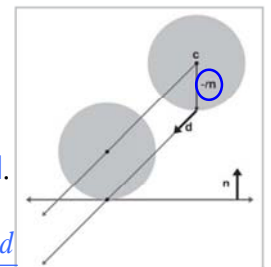
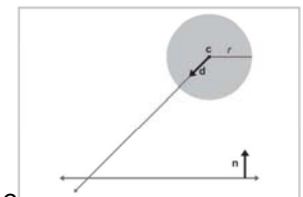
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Intersection of Sphere and Plane

- In dynamic situation, we will consider the plane to be stationary.
- We will define the plane using a normalized surface normal n and distance value d . $p \cdot n + d = 0$
- As t varies from $0 \dots l$, the motion of the center of the sphere is given by $c + td$, where c is the center of the sphere and d is a unit vector specifying the direction.
- Now, a ray defined by $p(t) = (c - m) + td$.
- We can give t as follows:

$$t = \frac{-(p_0 \cdot n + d)}{d \cdot n} = \frac{-((c - m) \cdot n + d)}{d \cdot n} = \frac{-c \cdot n + r - d}{d \cdot n}$$

Ray-plane intersection

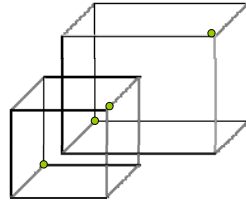


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Intersection of Two AABBs

- Detecting the intersection of two static AABBs is trivial.
 - Simply check for overlapping extents on each dimension independently.
 - Two AABBs intersect if they overlap on all three axes

```
# return true if all three axes overlap
xmin = max(b1.min[0], b2.min[0])
xmax = min(b1.max[0], b2.max[0])
ymin = max(b1.min[1], b2.min[1])
ymax = min(b1.max[1], b2.max[1])
zmin = max(b1.min[2], b2.min[2])
zmax = min(b1.max[2], b2.max[2])
return (xmin <= xmax) and (ymin <= ymax) and (zmin <= zmax)
```

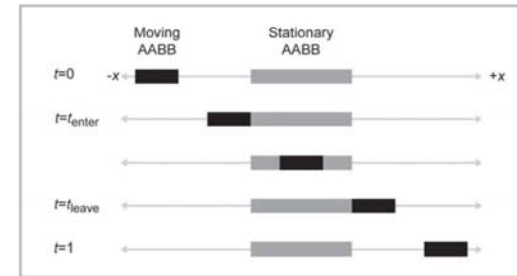


OR, exit with no intersection if separated along an axis

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Intersection of Two AABBs

- Dynamic intersection of AABBs
 - Consider a stationary AABB defined by extreme points s_{\min} and s_{\max} and a moving AABB defined by extreme points m_{\min} and m_{\max} and displacing by an amount given by the vector d , as t varies from 0...1.
 - Compute t , the parametric point in time where the moving box first collides with the stationary box.
 - First of all, we will solve the one-dimensional problem.



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Intersection of Two AABBs

- First of all, we will solve the one-dimensional problem.
 - A point t_{enter} where the boxes first begin to overlap
 - A point t_{leave} where the boxes cease to overlap

- Let $m_{\min}(t)$ and $m_{\max}(t)$ be the minimum and maximum values of the moving box at time t , given by:

$$m_{\min}(t) = m_{\min}(0) + td, \quad m_{\max}(t) = m_{\max}(0) + td$$

- Let s_{\min} and s_{\max} for the stationary box.

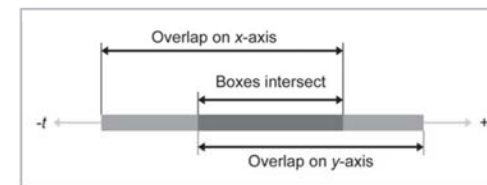
- Then, t_{enter} is the t value for which $m_{\max}(t)$ is equal to s_{\min} and t_{leave} is the t value for which $m_{\min}(t)$ is equal to s_{\max} .

$$t_{\text{enter}} = \frac{s_{\min} - m_{\max}(0)}{d} \quad t_{\text{leave}} = \frac{s_{\max} - m_{\min}(0)}{d}$$

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Intersection of Two AABBs

- In multi-dimensional problem, the intersection of these intervals on all dimensions gives the interval of time where the boxes intersect with each other.



- If the interval is empty, then the boxes never collide.
- If the interval lies completely outside the range $t=0...1$, then there is no collision over the period of time we are interested in.
- We will need to maintain the interval during which the boxes overlap, mainly to determine if it is empty.

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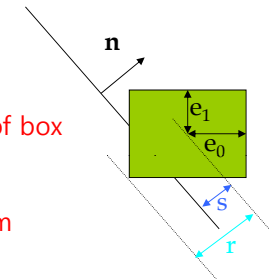
Intersection of AABB and Plane

- Let a AABBB defined by extreme points \mathbf{p}_{\min} and \mathbf{p}_{\max} and a plane defined by all points \mathbf{p} such that $\mathbf{p} \cdot \mathbf{n} + d = 0$.
- A simple static intersection test is to check whether the box is completely on one side of the plane or the other.
 - Actually, we don't have to check all eight corner points.
 - If $\mathbf{n}_x > 0$, then the corner with the minimum dot product has $x = x_{\min}$ and the corner with the maximum dot product has $x = x_{\max}$.
 - If $\mathbf{n}_x < 0$, then the opposite is true.
 - Similar statements apply to \mathbf{n}_y and \mathbf{n}_z .
 - Compute the minimum and maximum dot product values.
 - No intersection if (the minimum dot product value $> d$) or (the maximum dot product $< d$)
 - Otherwise, an intersection is detected.

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Intersection of AABB and Plane

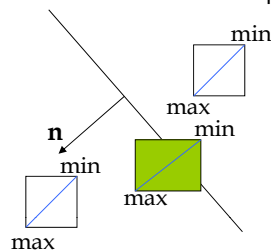
- # compute box center
- $c = (\max + \min) / 2$
- # compute positive extent length
- $e = \max - c$
- # compute the projection interval radius of box
- $r = e[0] * \text{abs}(n[0]) + e[1] * \text{abs}(n[1]) + e[2] * \text{abs}(n[2])$
- # compute the distance of box center from plane
- $s = \text{distance}(c, \text{plane})$
- # intersect if s is within $[-r, r]$ interval
- return $\text{abs}(s) \leq r$



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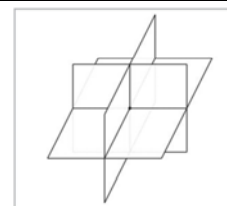
Intersection of AABBB and Plane

- For a dynamic test, consider the plane to be stationary.
 - Locate the corner points with the minimum and maximum dot product and move these points to the plane,
 - If AABBB is not initially intersecting the plane, it must strike the plane at the corner point closest to the plane.
 - For colliding with the "front" of the plane, we use the corner with the minimum dot product value



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Intersection of Three Planes



- In 3D, three planes intersect at a point.
- Let the three planes be defined implicitly as:

$$\mathbf{p} \cdot \mathbf{n}_1 + d_1 = 0 \quad \mathbf{p} \cdot \mathbf{n}_2 + d_2 = 0 \quad \mathbf{p} \cdot \mathbf{n}_3 + d_3 = 0$$

$$\mathbf{p} = \frac{-(d_1(\mathbf{n}_2 \times \mathbf{n}_3) + d_2(\mathbf{n}_3 \times \mathbf{n}_1) + d_3(\mathbf{n}_1 \times \mathbf{n}_2))}{(\mathbf{n}_1 \times \mathbf{n}_2) \cdot \mathbf{n}_3}$$

$$\mathbf{p} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^{-1} \begin{pmatrix} -d_1 \\ -d_2 \\ -d_3 \end{pmatrix} = \frac{1}{\det A} \text{adj}A \begin{pmatrix} -d_1 \\ -d_2 \\ -d_3 \end{pmatrix}$$

$$\det A = g(bf - ce) - h(af - cd) + i(ae - bd) = (\mathbf{n}_1 \times \mathbf{n}_2) \cdot \mathbf{n}_3$$

$$\text{adj}A = \begin{pmatrix} e_i - f_h & c_h - b_i & b_f - c_e \\ f_g - d_i & a_i - c_g & c_d - a_f \\ d_h - e_g & b_g - a_h & a_e - b_d \end{pmatrix}$$

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Other Tests

- ▣ Triangle-triangle intersection tests – many, many different ways to do this
- ▣ AABB-OBB (oriented bounding box) intersection tests
- ▣ Triangle-box intersection tests
- ▣ Tests using more “exotic” primitives, such as cylinders, cones, and tori