Game Physics

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Application in Video Games

- Racing games: Cars, snowboards, etc..
  - Simulates how cars drive, collide, rebound, flip, etc..
- Sports games
  - Simulates trajectory of soccer, basket balls.
- Increasing use in First Person Shooters: UnReal
  - Used to simulate bridges falling and breaking apart when blown up.
  - Dead bodies as they are dragged by a limb.
- Miscellaneous uses:
  - Flowing flags / cloth.
- Problem is that real time physics is very compute intensive. But it is becoming easier with faster CPUs.

Definitions

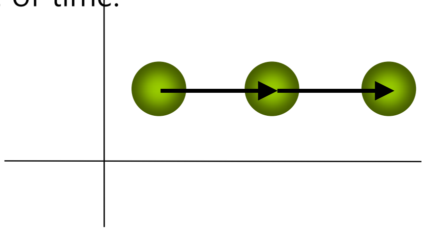
- Kinematics (운동학)
  - Study of movement over time.
  - Not concerned with the cause of the movement.
- Dynamics (동역학)
  - Study of forces and masses that cause the kinematic quantities to change as time progresses.

Game Physics

- Motion (운동)
- Position (위치), Velocity (속도), Acceleration (가속도)
- Force (힘), Gravity (중력)
- Buoyancy (부력), Drag (저항력)
- Friction (마찰력)
  - Kinetic friction (운동마찰)
  - Static friction (정지마찰)
- Spring (스프링)

Motion

- In physics, motion is a change in location or position of an object with respect to time.
- Object motion is represented with vectors
- Velocity is a vector
  - Vector direction is direction of movement
  - Vector magnitude is speed of movement
- Velocity vector corresponds to amount object will move in one unit of time.



Basic Motion

- Displacement (변위) = velocity * time
- If an object starts at position, P_0, with velocity v, after t time units, its position $P(t)$ is:

$$\mathrm{P}(t) = \mathrm{P}_0 + v\ t$$

- NOTE: choice of unit is arbitrary as long as things are consistent, e.g. meters for distance, seconds for time, meters/second for velocity

Varying Velocity

- The previous formula only works if the object moves with a constant velocity.
- In many cases, object's velocities change over time.
- The velocity is defined by the derivative:

$$v(t) = \frac{d}{dt}\mathrm{P}(t)$$

  - Constant velocity: $v(t) = v_0$
  - Velocity change over time by constant acceleration: $v(t) = v_0 + a\ t$
- The displacement is a function that we integrate velocity

$$displacement = \int_0^t velocity\ dt$$

Acceleration

- The acceleration is the rate of change in velocity.
- The acceleration is defined by the derivative

$$a(t) = \frac{d}{dt}v(t) = \frac{d^2}{dt^2}\mathrm{P}(t)$$

- Velocity is the integral of acceleration

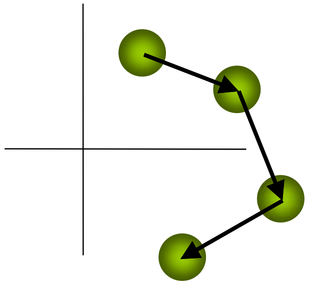
$$velocity = \int_0^t acceleration\ dt$$

Euler Method (or Euler Integration)

- Euler method (or Euler Integration) approximates an integral by step-wise addition.
  - The most basic kind of explicit method for numerical integration of ordinary differential equations (ODE).
- At each time step, we move the object in a straight line using the current velocity:

$$dt = t_1 - t_0$$

$$P(t_1) = P(t_0) + v\ dt$$



Euler Method (or Euler Integration)

- Applying Euler Integration to compute the position:

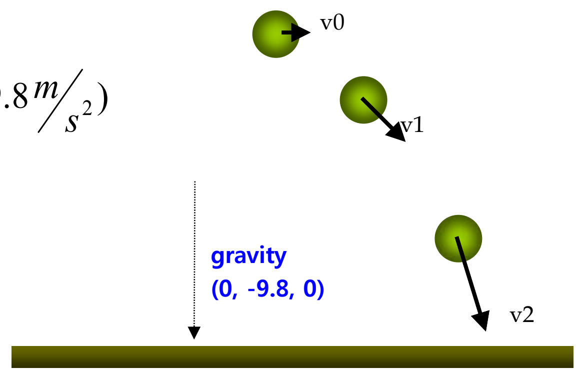
```
dt = t1 – t0;
Acc = ComputeAcceleration();
Vel = Vel + Acc * dt;
Pos = Pos + Vel * dt;
```

$$P(t) = \int_0^t (v_0 + at)dt = P_0 + v_0 t + \frac{1}{2} a\ t^2$$

Gravity

- Gravity (지구 중력 가속도) near the Earth's surface produces a constant acceleration of 9.8 meter/sec^2

$$F = mg\ \ (g = -9.8 \, {}^{m}\!/\!{}_{s^2})$$



Force

- Newton's second law of motion:

$$F = ma$$

$$\Rightarrow a = F / m$$

- If an object has mass M, and force F is applied to it, its motion can be calculated via Euler integration:

```
Acc = F/M;
Vel += Acc * dt;
Pos += Vel * dt;
```

Note that F, Acc, Vel, and Pos are all vectors. M is a scalar.

뉴튼 역학의 3법칙
- 관성의 법칙: 모든 물체는 다른 물체의 움직임의 영향을 받지 않는다고 할 때, 정지해 있었다면 계속 정지해 있을 것이고, 움직이고 있었다면 일정한 속도로 계속 운동할 것이다.
- 가속도의 법칙: 물체의 운동량의 변화율은, 크기와 방향에서, 그 물체에 작용하는 힘에 따른다.
- 작용, 반작용의 법칙: 모든 작용에는 그 반대방향으로 같은 크기의 반작용이 존재한다.

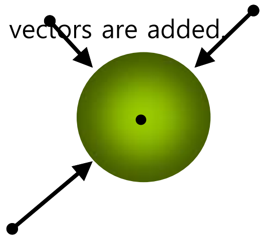
Gravitational Force

- Gravitational force (중력)
  - The force of attraction between all masses in the universe; especially the attraction of the earth's mass for bodies near its surface.
  - The gravitation between two bodies is proportional to the product of their masses and inversely proportional to the square of the distance between them.
- For a complete simulation, we need to calculate the force on each object every frame.
  - When multiple forces are applied, their vectors are added

$$F_{gravity} = \frac{GM_1M_2}{d^2} \quad (G = 6.673\times 10^{-11})$$

$$M_1, M_2 : mass(kg)$$

$$d : distance\ (meter)$$



Projectile Motion

- The projectile position, P_0, at t=0 with the velocity v_0:

$$P(t) = P_0 + v_0 t + \frac{1}{2}gt^2$$

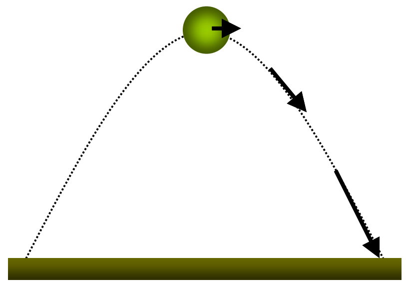
$$x(t) = x_0 + v_x t,\ \ y(t) = y_0 + v_y t - \frac{1}{2}gt^2,\ \ z(t) = z0 + v_z t$$

- Time to reach the maximum height, t:

$$y(t) = v_y t - gt^2 = 0 \Rightarrow t = \frac{v_y}{g}$$

- Maximum height, h:

$$h = y_0 + \frac{v_y^{\ 2}}{2g}$$



Projectile Motion

- Maximum range, r:

$$y(t) = y_0 + v_y t - \frac{1}{2}gt^2 = y_0 \Rightarrow t = 0\ or\ t = \frac{2v_y}{g}$$

$$x(t) = x_0 + v_x t \Rightarrow r = \frac{2v_x v_y}{g}$$

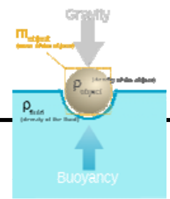
- Angle of elevation to reach the maximum height, θ:

$$h = y_0 + \frac{v_z^{\ 2}}{2g} \Rightarrow h = y_0 + \frac{(s\sin\theta)^2}{2g} \Rightarrow \theta = \sin^{-1}\left(\frac{1}{s}\sqrt{2g(h-y_0)}\right)$$

- Angle required to hit the target, θ:

$$r = \frac{2v_x v_y}{g} \Rightarrow r = \frac{2(s\cos\theta)(s\sin\theta)}{g} = \frac{s^2}{g}\sin 2\theta \Rightarrow \theta = \frac{1}{2}\sin^{-1}\frac{rg}{s^2}$$

Buoyancy



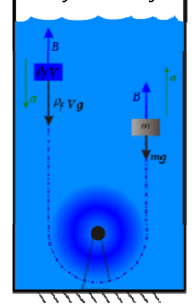
- Buoyancy force
  - Buoyancy is an upward acting force exerted by a fluid that oppose an object's weight.
  - Archimedes' principle:
    - Any floating object displaces its own weight of fluid.
    - I.e., any object, wholly or partially immersed in a fluid, is buoyed up by a force equal to the weight of the fluid displaced by the object.

$$F_{buoyancy} = -\rho_f V g$$

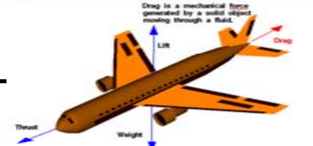
ρ_f is the density of the fluid

V is the volume of the displaced body of liquid

g is the gravitational acceleration



Drag



- Drag force
  - In fluid dynamics, drag refers to forces that oppose the relative motion of an object through a fluid (a liquid or gas).
  - Drag at low velocity (Stoke's drag):

$$F_d = -bv$$

$$b = 6\pi\eta r \ (\text{r : small spherical object radius}, \quad \eta : \text{viscosity})$$

  - Drag at high velocity:

$$F_d = \frac{1}{2}\rho v^2 A C_d (v \bullet v)\frac{v}{\|v\|}$$

F_d is the force vector of drag

ρ is the density of the fluid

v is the velocity of the object relative to the fluid

A is the reference area

C_d is the drag coefficien t

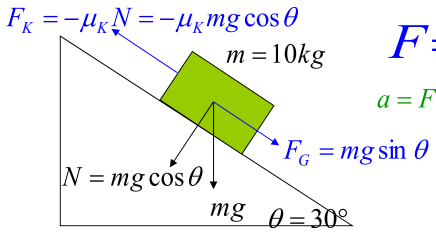
Kinetic Friction

- Kinetic friction
  - Kinetic (or dynamic) friction occurs when two objects are moving relative to each other and rub together (E.g., a sled on the ground).

$$F_K = -\mu_K N$$

N is the normal force

μ_K is the coefficien t of kinetic friction



$$F = F_G + F_K$$

$$a = F/m = (F_G + F_K)/m = g\sin\theta - \mu_K g\cos\theta$$

Static Friction

- Static friction
  - Static friction is the friction between two solid objects that are not moving relative to each other (E.g.., static friction can prevent an object from sliding down a sloped surface).

$$F_S = -\mu_S N$$

N is the normal force

μ_S is the coefficien t of static friction

  - The maximum value of static friction, F_{max}, when motion is impending, is sometimes referred to as limiting friction.
  - Any force larger than F_{max} overcomes the force of static friction and causes sliding to occur.

$$\mu_S mg\cos\theta = mg\sin\theta \Rightarrow \theta = \tan^{-1}\mu_S$$

Momentum

- Momentum, P, is the product of the mass and velocity of an object.
- The rate of change of the momentum of a particle is proportional to the resultant force acting on the particle and is in the direction of that force.

$$P = mv$$

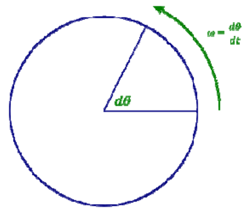
$$\Rightarrow \frac{dP}{dt} = m\frac{dv}{dt} = ma = F$$

```
Force = ComputeTotalForce();
Momentum += Force * dt;
Velocity = Momentum / Mass;
Position += Velocity* dt;
```

Angular Velocity

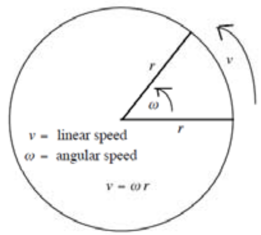
- Angular velocity (각속도) is the rate of change of angular displacement
- Angular velocity (radian/second):

$$\omega(t) = \frac{d}{dt}\theta(t)$$



- Relationship between angular velocity, ω, and linear velocity (선속도), v
  - Given a fixed speed v and radius r, then:

$$v(t) = \omega(t) \times r(t)$$



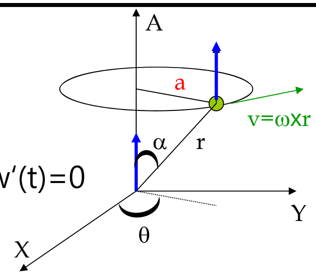
Centrifugal Force

- Linear acceleration (선가속도)

$$a(t) = \omega'(t) \times r(t) + \omega(t) \times r'(t)$$
$$= \omega'(t) \times r(t) + \omega(t) \times [\omega(t) \times r(t)]$$

- If the angular velocity is constant: w'(t)=0

$$a(t) = \omega(t) \times [\omega(t) \times r(t)]$$



  - Centrifugal force (원심력), equal and opposite to the tension (장력), drawing a rotating body away from the center of rotation.

$$F_c = -m(\omega(t) \times [\omega(t) \times r(t)])$$

  - Centrifugal force (when r(t) and w(t) is perpendicular):

$$F_c = m\omega^2 r = \frac{mv^2}{r}$$

Rigid Motion

- Rigid motion (강체운동)
  - A rigid body is an idealization of **solid body (e.g. car)** of finite size in which deformation is neglected. (only translation & rotation possible)
- Rigid body dynamics (강체동역학)
  - Linear & angular position, velocity, acceleration

```
Force = ComputeTotalForce();
Momentum += Force * dt;
Velocity = Momentum / Mass;
Position += Velocity* dt;
Torque = ComputeTotalTorque();
AngMomentum += Torque * dt;
Matrix I = Matrix*RotInertia*Matrix.Inverse(); // tensor
AngVelocity = I.Inverse()*AngMomentum;
Matrix.Rotate(AngVelocity*dt);
```

Integration Method

- Euler method
  - $v = v_0 + a*dt$
  - $P = P_0 + v*dt$

```
float t = 0;   // 현재 시간
float dt = 1; // 시간 간격 (timestamp)
float velocity = 0;      // 초기 속도
float position = 0;      // 초기 위치
float force = 10;
float mass = 1;
float acceleration = force/mass;
while (t<=10) {
    position += velocity * dt;
    velocity += acceleration * dt;
    t += dt;
}
```

Initial : $y'(t) = f(t, y(t)), y(t_0) = y_0$
Euler Method : $y_{n+1} = y_n + hf(t_n, y_n)$

Integration Method

- Runge-Kutta method

Initial : $y' = f(t, y),\ y(t_0) = y_0$

RK4 : $y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

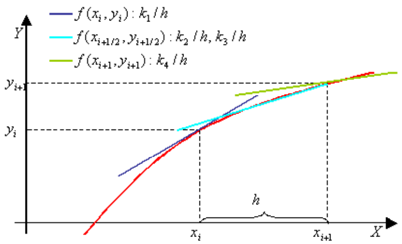
$k_1 = f(t_n, y_n)$

$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$

$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$

$k_4 = f(t_n + h, y_n + hk_3)$

$slope = \frac{k_1 + 2k_2 + 2k_3 + k_4}{6}$



Integration Method

```
void RK4Integration(vector3& pos, vector3& vel, float t, float dt) {
    vector3 k1Vel = vel;
    vector3 k1Acc = f(t, pos, vel);
    vector3 k2Vel = vel + 0.5f * dt * k1Acc;
    vector3 k2Acc = f(t + 0.5f * dt, pos + 0.5f * dt * k1Vel, k2Vel);
    vector3 k3Vel = vel + 0.5f * dt * k2Acc;
    vector3 k3Acc = f(t + 0.5f * dt, pos + 0.5f * dt * k2Vel, k3Vel);
    vector3 k4Vel = vel + dt * k3Acc;
    vector3 k4Acc = f(t + dt, pos + dt * k3Vel, k4Vel);
    pos += (dt / 6.0f) * (k1Vel + 2.0f * k2Vel + 2.0f * k3Vel + k4Vel);
    vel += (dt / 6.0f) * (k1Acc + 2.0f * k2Acc + 2.0f * k3Acc + k4Acc);
}
while (t<=10) {
    RK4Integration(position, velocity, t, dt);
    t += dt;
}
```

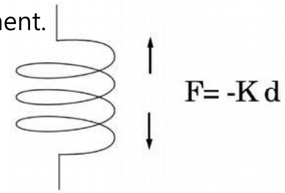
Springs

- Hooke's Law
  - Spring force is proportional to displacement.

$F = -K_s d$

K_s is the spring constant

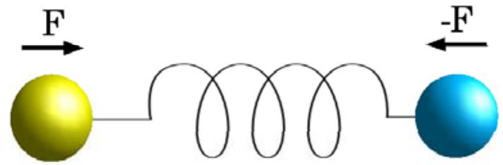
d is the displacement from rest length



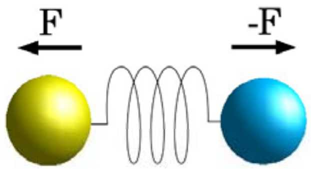
- Spring is modeled as two point masses, linked by the spring.
- Equal but opposite force is applied to each end.

Springs

- When spring is stretched, spring force pulls masses together.



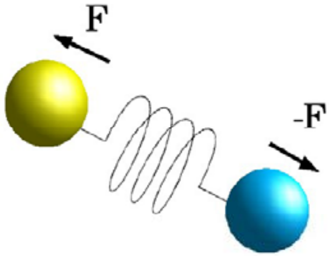
- When spring is compressed, spring force pushes masses apart.



Springs

- Vector between the points is used to compute displacement and the direction of force:

```
Vector3 v = point1 – point0;
float displacement = v.length() – restLength;
v.normalize();
Vector3 force = springConstant * displacement * v;
```



Spring Classes

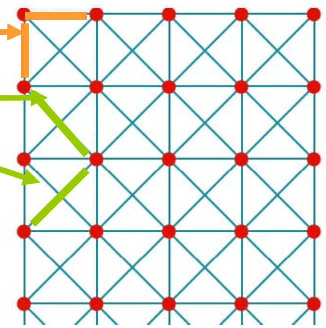
```
class PointMass
{
    float mass;
    float position[3];
    float velocity[3];
    float acceleration[3];
    void ClearForces();
    void AddForce();
    void Update();
    void Freeze();
}
```

Spring Classes

```
class Spring
{
    float pointMass[2];
    float springConstant;
    float restLength;
    void Update();
}
```

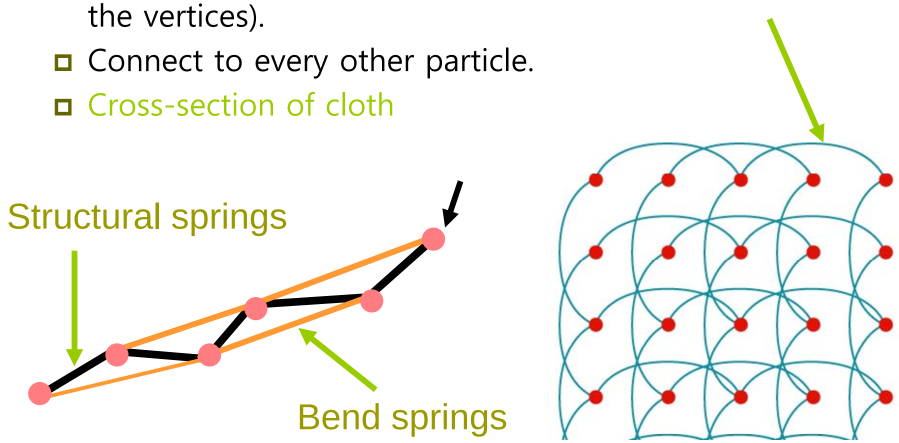
Simulating Cloth

- Cloth can be simulated by a mesh of springs.
- Structural Springs
- Shear Springs (to prevent the flag from shearing)



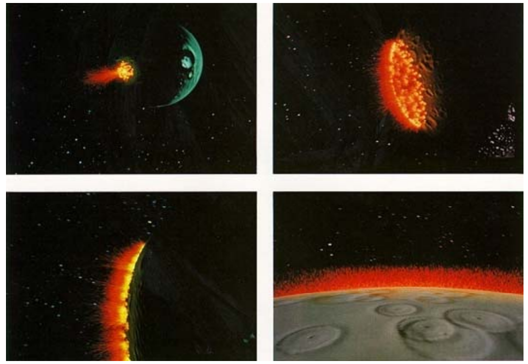
Simulating Cloth

- Bend Springs (to prevent the flag from folding along the vertices).
- Connect to every other particle.
- Cross-section of cloth



Particle Systems

- First used for graphics in Star Trek II (1983) "Genesis Effect"



Particle Systems

- Particle systems simulate explosions, smoke, fire, spray.
- They are also useful for modeling non-rigid objects such as jelly or cloth (more later).
- Infinitely small objects that have **Mass, Position and Velocity**
- Motion of a Newtonian particle is governed by:
  - F=ma (F=force, m=mass, a=acceleration)
  - a=dv/dt (Change of velocity over time- v=velocity; t=time)
  - v=dp/dt (Change of distance over time- p=distance or position)
  - So a basic data structure for a particle consists of: F, m, v, p.

E.g. a 3D particle might be represented as:

```
class Particle
{
    float mass;
    float position[3];// [3] for x,y,z components
    float velocity[3];
    float forceAccumulator[3];
}
```

- forceAccumulator is here because the particle may be acted upon by several forces- e.g. a soccerball is affected by the force of gravity and an external force like when someone kicks it. (see later)
- Anything that will impart a force on the particle will simply ADD their 3 force components (force in X,Y,Z) to the forceAccumulator.

E.g. 3D Particle System

```
class ParticleSystem
{
    particle *listOfParticles;
    int numParticles;
    void EulerStep();// Discussed later
}
```

Particle Dynamics Algorithm

For each particle

{

Compute the forces that are acting on the particle.

Compute the acceleration of each particle:

Since F=ma; a=F/m

Compute velocity of each particle due to the acceleration.

Compute the new position of the particle based on the velocity.

}

How do you calculate velocity?

- Recall that:
  - a = dv/dt (ie change in velocity over time)
  - v = dp/dt (ie change in position over time)
- So to find velocity we need to find the integral of acceleration
- To find the position we need to find the integral of velocity
- A simple numerical integration method (**Euler's Method**):
  - **Q(t+dt) = Q(t) + dt * Q'(t)**
  - So in our case:
    - To find velocity at each simulation timestep:
      - **v(t+dt) = v(t) + dt * v'(t) = v(t) + dt * a(t) // we know a(t) from F=ma**
    - To find the position at each simulation timestep:
      - **p(t+dt) = p(t) + dt * p'(t) = p(t) + dt * v(t) // we know v(t)**

E.g. Euler Integration EulerStep

- To find velocity at each simulation timestep:

  v(t+dt) = v(t) + dt * a(t) // we know a(t) from F=ma

  v_next[x] = v_now[x] + dt * a[x];

  v_next[y] = v_now[y] + dt * a[y];

  v_next[z] = v_now[z] + dt * a[z];
- To find the position at each simulation timestep:

  p(t+dt) = p(t) + dt * v(t) // we know v(t)

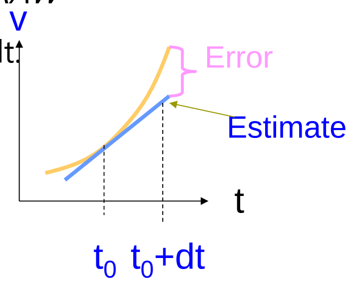
  p_next[x] = p_now[x] + dt * v_now[x];

  p_next[y] = p_now[y] + dt * v_now[y];

  p_next[z] = p_now[z] + dt * v_now[z];
- Remember to save away v_next for the next step through the simulation:
  - v_now[x] = v_next[x]; v_now[y] = v_next[y]; v_now[z] = v_next[z];

Warning about Euler Method

- Big time steps causes big integration errors.
- You know this has happened because your particles go out of control and fly off into infinity!
- Use small time steps- but note that small time steps chew up a lot of CPU cycles.
- You do not necessarily have to *DRAW* every time step. E.g. compute 10 timesteps and then draw the result.
- There are other better solutions:
  - Adaptive Euler Method
  - Midpoint Method
  - Implicit Euler Method
  - Runge Kutta Method



Adaptive Step Sizes

- Ideally we want the step-size (dt) to be as big as possible so we can do as few calculations as possible.
- But with bigger step sizes you incorporate more errors and your system can eventually destabilize.
- So small step sizes are usually needed. Unfortunately smaller step sizes can take a long time.
- You don't want to force a small step size all the time if possible.

Euler with Adaptive Step Sizes

- Suppose you compute 2 estimates for the velocity at time t+dt:
- So v1 is your velocity estimate for t+dt
- And v2 is your velocity estimate if you instead took 2 smaller steps of size dt/2 each.
- Both v1 and v2 differ from the true velocity by an order of dt^2 (because Euler's method is derived from Taylor's Theorem truncated after the 2nd term- see reference in the notes section of this slide)
- By that definition, v1 and v2 also differ from each other by an order of dt^2
- So we can write a measure of the current error as: E = |v1-v2|
- Let $E_{tolerated}$ be the error that YOU can tolerate in your game.
- Adaptive step size dt_{adapt} is calculated as approximately:

$$dt_{adapt} = Sqrt(E_{tolerated} / E) * dt$$

- So a bigger tolerated error would allow you to take a bigger step size. And a smaller one would force a smaller step size.

Handling Collisions

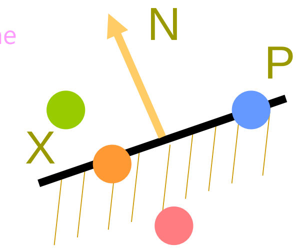
- Particles often bounce off surfaces.
  1. Need to detect when a collision has occurred.
  2. Need to determine the correct response to the collision.

Detecting Collision

- General Collision problem is complex:
  - Particle/Plane Collision – we will look at this one coz it's easy way to start
  - Plane/Plane Collision
  - Edge/Plane Collision

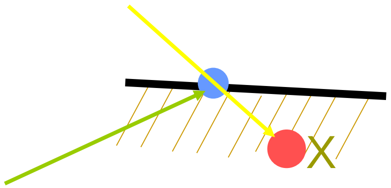
Particle/Plane Collisions

- P=any point on the plane
- N=normal pointing on the "legal" side of the plane.
- X=position of point we want to examine.
- For **(X – P) . N**
  - If > 0 then X is on legal side of plane.
  - If = 0 then X is on the plane.
  - If < 0 then X is on the wrong side of plane



Collision Response – dealing with the case where particle penetrates a plane (and it shouldn't have)

- If particle X is on the wrong side of the plane, move it to the surface of the plane and then compute its collision response.



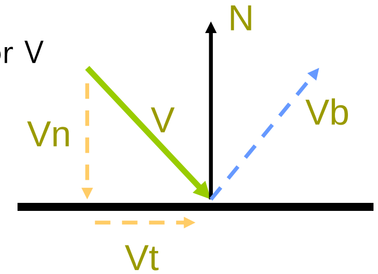
Collision Response

- N=normal to the collision plane
- Vn=normal component of a vector V is

  Vn= (N . V) V
- Vt=tangential component is:

  Vt=V-Vn
- Vb=bounced response:

  Vb=(1 – Kf) * Vt – (Kr * Vn)
- Kr=coefficient of restitution: ie how bouncy the surface is.
  1=perfectly elastic; 0=stick to wall.
- Kf=coefficient of friction: ie how much the tangential vector is slowed down after the bounce.
  1=particle stops in its tracks. 0=no friction.



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